KIRGIZİSTAN TÜRKİYE MANAS ÜNİVERSİTESİ
FEN BİLİMLERİ ENSTİTÜSÜ
MATEMATİK ANABİLİM DALI

$x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$ MAKSİMUMLU FARK DENKLEM SİSTEMİNİN ÇÖZÜMLERİ

Hazırlayan
Nurtilek Camşitov

Danışman
Doç. Dr. Dağıstan ŞİMŞEK

YÜKSEK LİSANS BİTİRME TEZİ

Haziran 2018
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BİLİMSEL ETİĞE UYGUNLUK

Bu çalışmadaki tüm bilgilerin, akademik ve etik kurallara uygun bir şekilde elde edildiğini beyan ederim. Aynı zamanda bu kural ve davranışların gerektirdiği gibi, bu çalışmanın özünde olmayan tüm materyal ve sonuçları tam olarak aktardığımı ve referans gösterdiğimi belirtirim.

Nurtilek Camşitov

İmza :

YÖNERGEYE UYGUNLUK

Nurtilek Camşitov adlı öğrenci Kırgızistan Türkiye Manas Üniversitesi Lisansüstü Eğitim-Öğretim ve Tez Hazırlama ve Yazma Yönergesi'ne uygun olarak hazırlanmıştır.

Tezi Hazırlayan

Nurtilek Camşitov

İmza:

Danışman

Doç. Dr. Dağıstan ŞİMŞEK

İmza:

Matematik ABD Başkanı

Prof.Dr. Anarkül URDALETOVA

İmza:

KABUL VE ONAY

Doç. Dr. Dağıstan ŞİMŞEK danışmanlığında Nurtilek Camşitov tarafından hazırlanan "$x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$ Maksimumlu Fark Denklem Sisteminin Çözümleri" adlı bu çalışma, jürimiz tarafından Kırgızistan Türkiye Manas Üniversitesi Fen Bilimleri Enstitüsü Matematik Anabilim Dalında Yüksek Lisans tezi olarak kabul edilmiştir.

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Doç. Dr. Dağıstan ŞİMŞEK
Enstitü Müdürü

КАБЫЛ АЛУУ ЖАНА БЕКИТҮҮ

Доц. Др. Дагыстан Шимшек жетекчилигинде Нуртилек Жамшитов тарабынан даярдалган “$x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$ максимумдуу айырма тендемелер системасын чыгаруу” аттуу темада магистрдик иш комиссия тарабынан Кыргыз-Түрк “Манас” университети Табигый илимдер институту, Математика бөлүмүнүн илимий багытында магистрдик иш болуп кабыл алынды.

……….. /…… / ………..

Комиссия:

Илимий Жетекчи : Доц. Док. Дагыстан Шимшек …………….

Төрагасы **:** ф-м. и. док., профессор Байзаков Асан …………….

Мүчө **:** ф-м. и. к., профессор Урдалетова Анаркүл …………….

Мүчө **:** ф-м. и. док., профессор Абдуллаев Фахреддин …………….

Мүчө **:** ф-м. и. док., профессор Өмүралиев Асан …………….

Мүчө **:** ф-м. и. док., профессор Асанов Авыт …………….

ЧЕЧИМ:
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………. /……../ ………

Доц. Др. Дагыстан Шимшек
Институт мүдүрү

ÖNSÖZ / TEŞEKKÜR

Çalışmalarım boyunca farklı bakış açıları ve bilimsel katkılarıyla beni aydınlatan, yakın ilgi ve yardımlarını esirgemeyen ve bugünlere gelmemde en büyük katkı sahibi sayın hocam Doç. Dr. Dağıstan ŞİMŞEK'e teşekkürü bir borç bilirim.

Ayrıca çalışmalarım süresince sabır göstererek beni daima destekleyen aileme en içten teşekkürlerimi sunarım.

Nurtilek Camşitov
Bişkek, Haziran 2018

$x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$ **MAKSİMUMLU FARK DENKLEM SİSTEMİNİN ÇÖZÜMLERİ**

Nurtilek CAMŞİTOV
Kırgızistan Türkiye Manas Üniversitesi, Fen Bilimleri Enstitüsü
Yüksek Lisans Tezi, Haziran 2018
Danışman: Doç. Dr. Dağıstan ŞİMŞEK

KISA ÖZET

Bu çalışma dört bölümden oluşmaktadır. Birinci bölümde, maksimumlu fark denklemleri ile ilgili bazı çalışmalar hakkında bilgi verdik. Bu bölümde kaynaklarda yer alan makale ve doktora tezlerinde yapılan çalışmalar hakkında kısa bilgiler verildi.

İkinci bölümde, maksimumlu fark denklemleri ile ilgili çalısmada kulanılan tanım ve teoremler verildi.

Üçüncü bölümde, maksimumlu fark denklem sisteminin çözümleri ve çözümlerin davranışları incelendi. İnceleme aşamasında 2 Lemma, 15 Teorem ve son olarakda örnekler verildi. Lemmalar da maksimumlu fark denklem sisteminin çözüm davranışları incelendi. Ayrıca; maksimumlu fark denklem sistemi için 7 farklı başlangıç değerine karşılık 7 tane örnek ve çözümleri verildi.

Dördüncü bölümde ise tez çalışmamız hakkındaki sonuç ve önerilere yer verildi.

Anahtar Kelimeler: Maksimumlu Fark Denklem Sistemi, Çözümlerin Davranışları, Periyodiklik

$x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$ МАКСИМУМДУУ АЙЫРМА ТЕНДЕМЕЛЕР СИСТЕМАСЫН ЧЫГАРУУ

Нуртилек ЖАМШИТОВ

Кыргыз-Түрк "Манас" университети, Табигый илимдер институту

Магистрдык иш, Июнь 2018

Илимий жетекчи: Доцент. Док. Дагыстан ШИМШЕК

АННОТАЦИЯ

Бул диссертация 4 бөлүмдөн турат. Биринчи бөлүмдө максимумдуу айырма тендемелер менен байланыштуу илимий иштер жөнүндө маалымат берилген. Бул бөлүмдө адабияттарда бар болгон макала жана доктордук диссертацияларда жасалган илимий иштер жөнүндө кыскача маалымат берилген.

Экинчи бөлүмдө, айырма тендемелер менен байланыштуу аныктама жана теория берилген.

Үчүнчү бөлүмдө максимумдуу айырма тендемелер системасынын чыгарылыштары изилденген. Изилдөө боюнча 2 Лемма, 15 Теорем жана мисалдер берилген. Леммаларда максимумдуу айырма тендемелер системасынын чыгарылыштары изилденген. Мындан сырткары максимумдуу айырма тендемелер системасы үчүн 7 башкача башталуу мааниси үчүн 7 мисал жана чыгарылыштары берилген.

Төртүнчү бөлүмдө болсо диссертация жөнүндө жыйынтык берилген.

Ачкыч сөздөр: Максимум айырма тендемеси, чыгарылыштарын изилдөөсү, мезгилдүү

$x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$ РЕШЕНИЯ СИСТЕМ РАЗНОСТНЫХ УРАВНЕНИЙ МАКСИМУМОВ

Нуртилек ЖАМШИТОВ

Кыргызско-Турецкий университет “Манас”, Институт Естественных наук

Магистерская работа, июнь 2018

Научный руководитель: Доц Док. Дагыстан ШИМШЕК

АННОТАЦИЯ

Эта диссертация состоит из 4 разделов. В первом разделе предоставляется информация о научных работах связанных с максимумами разностных уравнений. В этом разделе предоставлена краткая информация о проделанных работах из имеющихся литератур статей и докторских диссертациях.

Во втором разделе дано общих определений и теория связанная со разностными уравнениями.

В третьем разделе были исследованы решения систем максимумных разностных уравнений. По исследованиям даны 2 Леммы, 15 Теоремы и примеры. В леммах исследованы решения систем макмимумных разностных уравнений. Кроме этого, даны 7 разных началом значений 7 примера с решениями для системы максимумных разностных уравнений.

В четвертом разделе предоставлено заключение о диссертации.

Ключевые слова: Максима разностного уравнения системы, поведение решений, периодический

$x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$ SOLUTION OF THE SYSTEM OF MAXIMUM DIFFERENCE EQUATIONS

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M.Sc. Thesis, June 2018

Supervisor: Assoc.Prof. Dagıstan SHIMSHEK

ABSTRACT

This study consists of four parts. In the first section, we gave information about some of the work which are related to the maximal of difference equations. This resource is located in part of Article and were given a brief description of the work which was done in the doctoral thesis.

In the second part we give seven general definitions of difference equations and one theorem.

In the third section, we examine the solution of the maximal difference equations and their behavior. In this work we gave 2 Lemmas, 15 theorems and examples. In lemmas were examined behavior of solutions system of maximal difference equations.Also for system of maximal difference equations was given: For 7 different starting values 7 examples.

In the fourth part was given conclusions and recommendations of our thesis.

Keywords: System of maximal difference equations, Behavior of Solutions, periodicity

İÇİNDEKİLER

1. BÖLÜM

GİRİŞ

Bu çalışmada, maksimumlu fark denklem sisteminin çözümleri ve çözüm davranışları incelenmiştir.

Maksimumlu fark denklemleri ile ilgili literatürde var olan çalışmalardan büyük bir kısmı incelenmiştir. Bu kapsamlı araştırmanın ışığında, $A, x_0, x_{-1}, y_0, y_{-1}$ başlangıç şartları sıfırdan farklı reel sayılar olmak üzere,

$$x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\} \tag{1}$$

maksimumlu fark denklem sisteminin çözümleri ve çözüm davranışları incelenmiştir.

Öncelikle çalışmada kullanılan literatürün özeti ele alınmıştır [1-32].

1.1. Maksimumlu Fark Denklemleri İle İlgili Yapılmış Çalışmalar

A. M. Amleh (1998), yaptığı doktora tezinde, fark denklemlerin bazı periyodiklik ve kararlılık özelliklerini incelemiştir.

Amleh (1998), G. Ladas yönetiminde yaptığı doktora tezinde, fark denklemlerinin üç farklı konusunu ele almıştır. İlk bölümde, $x_{n+1} = \max\left\{\frac{A}{x_n}, \frac{B}{x_{n-1}}\right\}$ fark denkleminin çözüml erinin sıfırdan farklı reel sayılar olan A, B parametreleri ve x_{-1}, x_0 başlangıç şartları için periyodik olduğunu göstermiştir. İkinci bölümde, $x_{n+1} = \frac{x_n + x_{n-1}x_{n-2}}{x_n x_{n-1} + x_{n-2}}$ rasyonel fark denkleminin global asimptotik kararlılığını incelemiş ve son bölümde ise, Plant-Herbivore sisteminin çözümlerinin sınırlılığı üzerine çalışmıştır.

Janowski ve arkadaşları (1998), yaptıkları çalışmada; $x_{n+1} = \dfrac{\max\{x_n^k, A\}}{x_{n-1}}$ maksimumlu rasyonel fark denkleminin çözümlerinin sınırlılık ve salınımlılık özelliklerini incelemişlerdir. Bu fark denkleminde A, k parametreleri ve başlangıç şartlarının pozitif sayı değerleri aldıklarını varsaymışlar ve çalışma sonucunda bu denklemin çözümlerinin sınırlı ve salınımlı olma şartlarını A, k parametreleri ile başlangıç şartlarına bağlı olarak elde etmişlerdir.

S. Elaydi (1996), fark denklemine giriş kısmını incelemiştir.

Şimşek ve arkadaşları (2009), yaptıkları çalışmada; $x_{n+1} = \max\left\{\dfrac{1}{x_n}, \dfrac{y_n}{x_n}\right\}$, $y_{n+1} = \max\left\{\dfrac{1}{y_n}, \dfrac{x_n}{y_n}\right\}$, maksimumlu fark denklem sisteminin başlanğıç şartlarını pozitif seçerek çözümlerini incelemişlerdir.

Valicenti (1999), yaptığı doktora tezinde; $x_{n+1} = \dfrac{a_n x_n + b_n}{x_{n-1}}$ otonom olmayan Lyness fark denklemi ile $x_{n+1} = \dfrac{\max\{a_n x_n, b_n\}}{x_{n-1}}$ maksimumlu fark denkleminin çözümlerinin periyodikliği ve global asimptotik kararlılığı üzerine çalışmıştır.

Teixeria (2000), yaptığı doktora tezinde; ilk olarak A herhangi bir reel sayı ve başlangıç şartları sıfır olmayan reel sayılar olmak üzere, $x_{n+1} = \dfrac{\max\{x_n, A\}}{x_n x_{n-1}}$ fark denkleminin çözümlerinin periyodikliğini incelemiştir. Daha sonra, $x_{n+1} = \dfrac{a}{x_n} + \dfrac{b}{y_n}$, $y_{n+1} = \dfrac{c}{x_n} + \dfrac{d}{y_n}$ fark denklem sisteminin çözümlerini analiz etmiş ve son olarak ta $y_{n+1} = \dfrac{p + y_{n-1}}{q y_n + y_{n-1}}$ fark denkleminin pozitif parametreler ve başlangıç şartları altında global asimptotik kararlı olduğunu göstermiştir.

Simsek ve arkadaşları (2006), yaptıkları çalışmada; $x_{n+1} = \max\left\{ \frac{1}{x_{n-1}}, x_{n-1} \right\}$ fark denkleminin pozitif başlangıç şartları altında çözümlerinin periyodikliğini incelemişlerdir.

Papaschinopoulos ve Hatzifilippidis (2001), katsayılarını pozitif sayı dizileri ve başlangıç şartlarını pozitif sayı olarak aldıkları $x_{n+1} = \frac{\max\left\{ a_n \left(\prod_{i=n-k+1}^{n} x_i \right), b_n \right\}}{\prod_{i=n-k}^{n} x_i}$ fark denkleminin pozitif çözümlerinin süreklilik, sınırlılık ve periyodiklik özelliklerini incelemişlerdir.

Mishev ve arkadaşları (2002), $x_{n+1} = \max\left\{ \frac{A}{x_n}, \frac{B}{x_{n-2}} \right\}$ fark denkleminin periyodikliği üzerine yaptıkları çalışmada; A, B parametreleri ile başlangıç şartlarını pozitif sayı değerleri olarak kabul ederek denklemin bütün pozitif çözümlerinin er geç periyodik olduğunu ispat etmişlerdir.

Voulov (2002), yaptığı iki çalışmadan birincisinde; G. Ladas tarafından verilen bir açık problemi çözmüştür. Bu çalışmada, A, B, C parametreleri negatif olmayan reel sayılar olmak üzere $A + B + C > 0$ için $x_n = \max\left\{ \frac{A}{x_{n-1}}, \frac{B}{x_{n-3}}, \frac{C}{x_{n-5}} \right\}$ fark denkleminin bütün çözümlerinin periyodik olduğunu göstermiştir. İkincisinde ise, A ile B parametreleri pozitif reel sayılar ve k ile m parametreleri pozitif tam sayılar olmak üzere, $x_{n+1} = \max\left\{ \frac{A}{x_{n-k}}, \frac{B}{x_{n-m}} \right\}$ maksimumlu fark denkleminin pozitif çözümlerinin periyodiklik özelliğini incelemiştir. A, B, k ve m parametrelerine bağlı olarak denklemin bütün pozitif çözümlerinin er geç periyodik olduğunu ispat etmiştir.

Papaschinopoulos ve arkadaşları (2003), yaptıkları çalışmada daha önce Feuer tarafından çalışılmış olan $x_{n+1} = \dfrac{\max\{x_n, A\}}{x_n x_{n-1}}$ fark denkleminin çözümleri, çözümlerinin periyodikliği ve sabit aralığı üzerine çalışmışlardır.

Feuer (2003), $x_{n+1} = \dfrac{\max\{x_n^k, A\}}{x_n^l x_{n-1}}$ maksimumlu Lyness fark denklemi üzerinde yaptığı çalışmada; A 'nın pozitif bir reel sayı, k, l ve başlangıç şartlarının da keyfi reel sayı değerleri olduğunu kabul ederek denklemin çözümlerinin periyodiklik özelliğini incelemiştir.

Patula ve Voulov (2004), yaptıkları çalışmada; A_n, B_n pozitif terimli ve 3 periyotlu diziler olmak üzere, $x_{n+1} = \max\left\{\dfrac{A_n}{x_n}, \dfrac{B_n}{x_{n-2}}\right\}$ fark denkleminin çözümlerinin periyodikliğini incelemişlerdir.

Çinar ve arkadaşları (2005), yaptıkları çalışmada; $A, B > 0$ olmak üzere, sıfırdan farklı başlangıç şartları için $x_{n+1} = \min\left\{\dfrac{A}{x_n}, \dfrac{B}{x_{n-2}}\right\}$ fark denkleminin pozitif çözümlerinin periyodikliğini incelemişlerdir. Ayrıca, bu denklemi genelleştirerek elde ettikleri $x_{n+1} = \min\left\{\dfrac{A}{x_n x_{n-1} \dots x_{n-k}}, \dfrac{B}{x_{n(k+2)} \dots x_{n-(2k+2)}}\right\}$ fark denkleminin pozitif çözümlerinin periyodikliğini incelemişlerdir.

Şimşek (2007), Bazı Fark Denklemlerinin Çözümleri ve Periyodikliği Üzerine Bir Çalışma adlı doktora tezinde maksimumlu fark denkleminin çözümlerini incelemişdir.

Şimşek ve arkadaşları (2009), yaptıkları çalışmada $x_{n+1} = \max\left\{\frac{A}{x_n}, \frac{y_n}{x_n}\right\}$, $y_{n+1} = \max\left\{\frac{A}{y_n}, \frac{x_n}{y_n}\right\}$, maksimumlu fark denklem sisteminde A'yı başlangıç şartlarını pozitif seçerek çözümünü incelemişlerdir

Yan ve arkadaşları (2006), yaptıkları çalışmada; $0 < \alpha < 1, A > 0, A \leq 1, \quad A > 1$ ve $x_{-2}, x_{-1}, x_0 \in (0, \infty)$ başlangıç şartları için $x_n = \max\left\{\frac{1}{x_{n-1}^{\alpha}}, \frac{A}{x_{n-2}}\right\}$ fark denkleminin çözümlerinin 4 periyotlu olduğunu göstermişlerdir.

Fark denklemlerinin yeni çalışma alanlarından olan maksimumlu fark denklemleri ile ilgili literatürde son yıllarda yapılmış oldukça fazla sayıda çalışma vardır.

2. BÖLÜM

2.1.MAKSİMUMLU FARK DENKLEMLERİ İLE İLGİLİ ÇALIŞMADA KULANILAN TANIM VE TEOREMLER

Bu bölümde maksimumlu fark denklemleri ile ilgili literatürde var olan ve tezde kullanılan genel tanım ve teoremler verilmiştir [1-32].

x bağımsız değişkeninin sürekli olduğu durumlarda, $y(x)$ bağımlı değişkeninin değişimi $y^{'}(x), y^{''}(x), ..., y^{(n)}(x), ...$ türevleri yardımıyla açıklanabilmektedir. Ancak x 'in kesikli değerler alması durumunda değişim türevler yardımıyla açıklanamaz. Bu bölümde x 'in tamsayı değerler aldığı durumlarda ortaya çıkan ve içinde sonlu farkların bulunduğu denklemler üzerinde duracağız.

Tanım 2.1. n bağımsız değişken ve buna bağımlı değişkende y olmak üzere, bağımlı değişken ve bağımsız değişken ile bağımlı değişkenin $E(y), E^2(y)$, $E^3(y), ..., E^n(y), ...$ gibi farklarını içeren bağıntılara Fark Denklemi denir. Dikkat edilirse, fark denklmlerinin n 'in sürekli olduğu durumda diferansiyel denklemler ile arasında büyük benzerlikler vardır.

Birinci mertebeden fark denklemi;

$$a_0 y(n) + a_1 y(n+1) = f(n)$$

şeklindedir.

İkinci mertebeden fark denklemi;

$$a_0 y(n-1) + a_1 y(n) + a_2 y(n+1) = g(n)$$

şeklindedir. Denklemin mertebesinin belirlenmesinde, y 'nin hesaplanabilmesi için gerekli olan başlangıç şartı sayısı göz önüne alınmaktadır.

Teorem 2.1. I reel sayıların herhangi bir alt aralığı olmak üzere, $f: I\, x\, I \to I$ sürekli diferensiyellenebilen bir fonksiyon olsun. Her $x_{-1}, x_0 \in I$ başlangıç şartları için

$$x_{n+1} = f(x_n, x_{n-1}), \quad n = 0,1,2,... \tag{2}$$

denklemi bir tek $\{x_n\}_{n=-1}^{\infty}$ çözümüne sahiptir.

Tanım 2.2. Eğer $\{x_n\}$ dizisi için $x_{n+p} = x_n$ ise, $\{x_n\}$ dizisi p periyotludur denir ve p bu şartı sağlayan en küçük pozitif tam sayıdır.

Tanım 2.3. Eğer $\{x_n\}$ dizisinde sonlu sayıda terim hariç tutulduğunda, geriye kalan sonsuz sayıdaki terim için $x_{n+p} = x_n$ ise, $\{x_n\}$ dizisine er geç p periyotludur denir ve p bu şartı sağlayan en küçük pozitif tam sayıdır.

Tanım 1 : $x_{n+1} = f(x_n,..., x_{n-k})$ n=0,1,2,... (2) fark denkleminde $\bar{x} = f(\bar{x},..., \bar{x})$ oluyorsa $\bar{x}$ ye denge noktası denir.

Tanım 2 : $\bar{x}$, (2) denkleminin pozitif bir denge noktası olsun. (2) denkleminin bir $\{x_n\}$ çözümünün bir pozitif yarı dönmesi $\{x_l, x_{l+1},..., x_m\}$ terimlerinin bir dizisinden oluşur ve bunların hepsi $\bar{x}$ denge noktasına eşit veya büyük bütün terimlerdir. Öyle ki $l \geq 0$ ve $\mathrm{m} \leq \infty$ olur ve burada

Ya $l = 0$ yada $l > 0$ ve $x_{l-1} < \bar{x}$ ve Ya

$m = \infty$ yada $m < \infty$ ve $x_{m+1} < \bar{x}$ dir.

Tanım 3 : $\bar{x}$, (2) denkleminin negatif bir denge noktası olsun. (2) denkleminin bir $\{x_n\}$ çözümünün bir negatif yarı dönmesi $\{x_l, x_{l+1},..., x_m\}$ terimlerinin bir dizisinden oluşur ve bunların hepsi $\bar{x}$ denge noktasından daha küçük terimlerdir. Öyle ki $l \geq 0$ ve $\mathrm{m} \leq \infty$ olur ve burada

Ya $l = 0$ yada $l > 0$ ve $x_{l-1} \geq \bar{x}$ ve

Ya $m = \infty$ yada $m < \infty$ ve $x_{m+1} \geq \bar{x}$ dir.

Tanım 4 : $f_1 = 1, f_2 = 1$ ve $\mathrm{n} \geq 3$ için $f_n = f_{n-1} + f_{n-2}$ şeklinde tanımlanan sayılara Fibonacci sayıları denir.

3. BÖLÜM

3.1.MAKSİMUMLU FARK DENKLEM SİSTEMİNİN ÇÖZÜMÜ

Şimdi (1) denkleminin pozitif denge noktasını bulalım.

$$\bar{x} = \max\left\{\frac{A}{\bar{x}}, \frac{\bar{y}}{\bar{x}}\right\}; \bar{y} = \max\left\{\frac{A}{\bar{y}}, \frac{\bar{x}}{\bar{y}}\right\}$$

$$\bar{x} = \frac{A}{\bar{x}} \text{ veya } \bar{x} = \frac{\bar{y}}{\bar{x}}; \bar{y} = \frac{A}{\bar{y}} \text{ veya } \bar{y} = \frac{\bar{x}}{\bar{y}},\ (\bar{x})^2 = A \text{ ve } (\bar{y})^2 = A$$

bulunur.

Lemma 1 : A=1

$$0 < x_{-1} < x_0 < y_{-1} < y_0 < A, 0 < x_{-1} < x_0 < y_0 < y_{-1} < A, 0 < x_{-1} < y_{-1} < x_0 < y_0 < A,$$
$$0 < x_{-1} < y_{-1} < y_0 < x_0 < A, 0 < x_{-1} < y_0 < y_{-1} < x_0 < A, 0 < x_{-1} < y_0 < x_0 < y_{-1} < A,$$
$$0 < y_0 < x_0 < y_{-1} < x_{-1} < A, 0 < y_0 < x_0 < x_{-1} < y_{-1} < A, 0 < y_0 < y_{-1} < x_0 < x_{-1} < A,$$
$$0 < y_0 < y_{-1} < x_{-1} < x_0 < A, 0 < y_0 < x_{-1} < y_{-1} < x_0 < A, 0 < y_0 < x_{-1} < x_0 < y_{-1} < A,$$
$$0 < y_{-1} < x_0 < y_0 < x_{-1} < A, 0 < y_{-1} < y_0 < x_0 < x_{-1} < A, 0 < y_{-1} < y_0 < x_{-1} < x_0 < A,$$
$$0 < y_{-1} < x_{-1} < y_0 < x_0 < A, 0 < y_{-1} < x_0 < x_{-1} < y_0 < A, 0 < y_{-1} < x_{-1} < x_0 < y_0 < A,$$
$$0 < x_0 < y_{-1} < y_0 < x_{-1} < A, 0 < x_0 < y_0 < x_{-1} < y_{-1} < A, 0 < x_0 < y_0 < y_{-1} < x_{-1} < A,$$
$$0 < x_0 < x_{-1} < y_0 < y_{-1} < A, 0 < x_0 < x_{-1} < y_{-1} < y_0 < A, 0 < x_0 < y_{-1} < x_{-1} < y_0 < A$$

Yukarıdaki başlangıç şartları için aşağıdakiler doğrudur :

$n \geq 0$ için x_n çözümleri ve $n \geq 0$ için y_n çözümlerinde

a) Her pozitif yarı dönme üç terimden oluşur.
b) Her negatif yarı dönme iki terimden oluşur.
c) Üç uzunluğundaki her positif yarı dönmeyi iki uzunluğundaki negatif yarı dönme takip eder.
d) İki uzunluğundaki her negatif yarı dönmeyi üç uzunluğundaki pozitif yarı dönme takip eder.

İspat :

$$x_1 = \max\left\{\frac{1}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \frac{1}{x_{-1}} > \bar{x} \qquad y_1 = \max\left\{\frac{1}{y_{-1}}, \frac{x_0}{y_{-1}}\right\} = \frac{1}{y_{-1}} > \bar{y}$$

$$x_2 = \max\left\{\frac{1}{x_0}, \frac{y_1}{x_0}\right\} = \max\left\{\frac{1}{x_0}, \frac{1}{x_0 y_{-1}}\right\} = \frac{1}{x_0 y_{-1}} > \bar{x} \qquad y_2 = \max\left\{\frac{1}{y_0}, \frac{x_1}{y_0}\right\} = \max\left\{\frac{1}{y_0}, \frac{1}{y_0 x_{-1}}\right\} = \frac{1}{y_0 x_{-1}} > \bar{y}$$

$$x_3 = \max\left\{\frac{1}{x_1}, \frac{y_2}{x_1}\right\} = \max\left\{x_{-1}, \frac{1}{y_0}\right\} = \frac{1}{y_0} > \bar{x} \qquad y_3 = \max\left\{\frac{1}{y_1}, \frac{x_2}{y_1}\right\} = \max\left\{y_{-1}, \frac{1}{x_0}\right\} = \frac{1}{x_0} > \bar{y}$$

$$x_4 = \max\left\{\frac{1}{x_2}, \frac{y_3}{x_2}\right\} = \max\left\{x_0 y_{-1}, y_{-1}\right\} = y_{-1} < \bar{x} \qquad y_4 = \max\left\{\frac{1}{y_2}, \frac{x_3}{y_2}\right\} = \max\left\{y_0 x_{-1}, x_{-1}\right\} = x_{-1} < \bar{y}$$

$$x_5 = \max\left\{\frac{1}{x_3},\frac{y_4}{x_3}\right\} = \max\{y_0, y_0x_{-1}\} = y_0 < \bar{x} \quad y_5 = \max\left\{\frac{1}{y_3},\frac{x_4}{y_3}\right\} = \max\{x_0, x_0y_{-1}\} = x_0 < \bar{y}$$

$$x_6 = \max\left\{\frac{1}{x_4},\frac{y_5}{x_4}\right\} = \max\left\{\frac{1}{y_{-1}},\frac{x_0}{y_{-1}}\right\} = \frac{1}{y_{-1}} > \bar{x} \quad y_6 = \max\left\{\frac{1}{y_4},\frac{x_5}{y_4}\right\} = \max\left\{\frac{1}{x_{-1}},\frac{y_0}{x_{-1}}\right\} = \frac{1}{x_{-1}} > \bar{y}$$

$$x_7 = \max\left\{\frac{1}{x_5},\frac{y_6}{x_5}\right\} = \max\left\{\frac{1}{y_0},\frac{1}{y_0x_{-1}}\right\} = \frac{1}{y_0x_{-1}} > \bar{x} \quad y_7 = \max\left\{\frac{1}{y_5},\frac{x_6}{y_5}\right\} = \max\left\{\frac{1}{x_0},\frac{1}{x_0y_{-1}}\right\} = \frac{1}{x_0y_{-1}} > \bar{y}$$

$$x_8 = \max\left\{\frac{1}{x_6},\frac{y_7}{x_6}\right\} = \max\left\{y_{-1},\frac{1}{x_0}\right\} = \frac{1}{x_0} > \bar{x} \quad y_8 = \max\left\{\frac{1}{y_6},\frac{x_7}{y_6}\right\} = \max\left\{x_{-1},\frac{1}{y_0}\right\} = \frac{1}{y_0} > \bar{y}$$

$$x_9 = \max\left\{\frac{1}{x_7},\frac{y_8}{x_7}\right\} = \max\{x_{-1}y_0, x_{-1}\} = x_{-1} < \bar{x} \quad y_9 = \max\left\{\frac{1}{y_7},\frac{x_8}{y_7}\right\} = \max\{x_0y_{-1}, y_{-1}\} = y_{-1} < \bar{y}$$

$$x_{10} = \max\left\{\frac{1}{x_8},\frac{y_9}{x_8}\right\} = \max\{x_0, x_0y_{-1}\} = x_0 < \bar{x} \quad y_{10} = \max\left\{\frac{1}{y_8},\frac{x_9}{y_8}\right\} = \max\{y_0, x_{-1}y_0\} = y_0 < \bar{y}$$

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$x_1 > \bar{x}$, $x_2 > \bar{x}$, $x_3 > \bar{x}$, $x_4 < \bar{x}$, $x_5 < \bar{x}$, $x_6 > \bar{x}$, $x_7 > \bar{x}$, $x_8 > \bar{x}$, $x_9 < \bar{x}$, $x_{10} < \bar{x}$, …

görüldüğü gibi x_n çözümleri PPPNNPPPNN… şeklinde devam eder.

$y_1 > \bar{y}$, $y_2 > \bar{y}$, $y_3 > \bar{y}$, $y_4 < \bar{y}$, $y_5 < \bar{y}$, $y_6 > \bar{y}$, $y_7 > \bar{y}$, $y_8 > \bar{y}$, $y_9 < \bar{y}$, $y_{10} < \bar{y}$, …

Buradan görüldüğü gibi y_n çözümleri PPPNNPPPNN… şeklinde devam eder.

Görüldüğü üzere $n \geq 0$ için x_n çözümleri ve $n \geq 0$ için y_n çözümlerinde; her negatif yarı dönme iki terimden oluşur. Her pozitif yarı dönme üç terimden oluşur. İki uzunluğundaki her negatif yarı dönmeyi üç uzunluğundaki pozitif yarı dönme takip eder. Üç uzunluğundaki her pozitif yarı dönmeyi iki uzunluğundaki negatif yarı dönme takip eder.

Lemma 2: Eğer A<1 ise $x_{n+1} = \max\left\{\frac{A}{x_{n-1}},\frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}},\frac{x_n}{y_{n-1}}\right\}$

denkleminin $(x_n; y_n)$ çözümleri :

$A < x_0 < y_{-1} < y_0 < x_{-1} < 1$, $A < x_0 < y_0 < x_{-1} < y_{-1} < 1$, $A < x_0 < y_0 < y_{-1} < x_{-1} < 1$,

$A < y_0 < x_{-1} < x_0 < y_{-1} < 1$, $A < y_0 < x_0 < y_{-1} < x_{-1} < 1$, $A < x_{-1} < x_0 < y_{-1} < y_0 < 1$,

$A < x_{-1} < x_0 < y_0 < y_{-1} < 1, A < x_{-1} < y_0 < x_0 < y_{-1} < 1, A < x_0 < x_{-1} < y_{-1} < y_0 < 1,$

$A < x_0 < x_{-1} < y_0 < y_{-1} < 1, A < x_0 < y_{-1} < x_{-1} < y_0 < 1, A < x_{-1} < y_{-1} < x_0 < y_0 < 1,$

$A < x_{-1} < y_{-1} < y_0 < x_0 < 1, A < x_{-1} < y_0 < y_{-1} < x_0 < 1, A < y_{-1} < x_{-1} < x_0 < y_0 < 1,$

$A < y_{-1} < x_{-1} < y_0 < x_0 < 1, A < y_{-1} < x_0 < x_{-1} < y_0 < 1, A < y_0 < x_{-1} < y_{-1} < x_0 < 1,$

$A < y_0 < y_{-1} < x_{-1} < x_0 < 1, A < y_0 < y_{-1} < x_0 < x_{-1} < 1, A < y_{-1} < x_0 < y_0 < x_{-1} < 1,$

$A < y_{-1} < y_0 < x_{-1} < x_0 < 1, A < y_{-1} < y_0 < x_0 < x_{-1} < 1, A < y_0 < x_0 < x_{-1} < y_{-1} < 1,$

Yukarıdaki başlangıç şartları için aşağıdakiler doğrudur :

$n \geq 0$ için x_n çözümleri ve $n \geq 0$ için y_n çözümlerinde

a) Her pozitif yarı dönme üç terimden oluşur.

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d) Üç uzunluğundaki her negatif yarı dönmeyi üç uzunluğundaki pozitif yarı dönme takip eder.

İspat:

Bu lemmanın ispatını n nin değerleri için gösterelim.

$$x_1 = \max\left\{\frac{A}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \frac{y_0}{x_{-1}} > \bar{x} \qquad y_1 = \max\left\{\frac{A}{y_{-1}}, \frac{x_0}{y_{-1}}\right\} = \frac{y_0}{x_{-1}} > \bar{y}$$

$$x_2 = \max\left\{\frac{A}{x_0}, \frac{y_1}{x_0}\right\} = \max\left\{\frac{A}{x_0}, \frac{1}{y_{-1}}\right\} = \frac{1}{y_{-1}} > \bar{x} \qquad y_2 = \max\left\{\frac{A}{y_0}, \frac{x_1}{y_0}\right\} = \max\left\{\frac{A}{y_0}, \frac{1}{x_{-1}}\right\} = \frac{1}{x_{-1}} > \bar{y}$$

$$x_3 = \max\left\{\frac{A}{x_1}, \frac{y_2}{x_1}\right\} = \max\left\{\frac{Ax_{-1}}{y_0}, \frac{1}{y_0}\right\} = \frac{1}{y_0} > \bar{x} \qquad y_3 = \max\left\{\frac{1}{y_1}, \frac{x_2}{y_1}\right\} = \max\left\{\frac{Ay_{-1}}{x_0}, \frac{1}{x_0}\right\} = \frac{1}{x_0} > \bar{y}$$

$$x_4 = \max\left\{\frac{1}{x_2}, \frac{y_3}{x_2}\right\} = \max\left\{\mathrm{A}\, y_{-1}, \frac{y_{-1}}{x_0}\right\} = \frac{y_{-1}}{x_0} < \bar{x} \qquad y_4 = \max\left\{\frac{A}{y_2}, \frac{x_3}{y_2}\right\} = \max\left\{\mathrm{A}\, x_{-1}, \frac{x_{-1}}{y_0}\right\} = \frac{x_{-1}}{y_0} < \bar{y}$$

$$x_5 = \max\left\{\frac{A}{x_3}, \frac{y_4}{x_3}\right\} = \max\left\{\mathrm{A}\, y_0, x_{-1}\right\} = x_{-1} < \bar{x} \qquad y_5 = \max\left\{\frac{A}{y_3}, \frac{x_4}{y_3}\right\} = \max\left\{\mathrm{A}\, x_0, y_{-1}\right\} = y_{-1} < \bar{y}$$

$$x_6 = \max\left\{\frac{A}{x_4}, \frac{y_5}{x_4}\right\} = \max\left\{\frac{Ax_0}{y_{-1}}, x_0\right\} = x_0 < \bar{x} \qquad y_6 = \max\left\{\frac{A}{y_4}, \frac{x_5}{y_4}\right\} = \max\left\{\frac{Ay_0}{x_{-1}}, y_0\right\} = y_0 < \bar{y}$$

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$x_1 > \bar{x},\ x_2 > \bar{x},\ x_3 > \bar{x},\ x_4 < \bar{x},\ x_5 < \bar{x},\ x_6 < \bar{x} \ldots$

görüldüğü gibi x_n çözümleri PPPNNN… şeklinde devam eder.

$y_1 > \bar{y},\ y_2 > \bar{y},\ y_3 > \bar{y},\ y_4 < \bar{y},\ y_5 < \bar{y},\ y_6 < \bar{y} \ldots$

Buradan görüldüğü gibi y_n çözümleri PPPNNN… şeklinde devam eder.

Görüldüğü üzere $n \geq 0$ için x_n çözümleri ve $n \geq 0$ için y_n çözümlerinde; her negatif yarı dönme üç terimden oluşur. Her pozitif yarı dönme üç terimden oluşur.Üç uzunluğundaki her negatif yarı dönmeyi üç uzunluğundaki pozitif yarı dönme takip eder. Üç uzunluğundaki her pozitif yarı dönmeyi üç uzunluğundaki negatif yarı dönme takip eder.

Teorem 1: Eğer A=1 ise $x_{n+1} = \max\left\{\frac{1}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{1}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$

denkleminin $(x_n; y_n)$ çözümleri

$$0<x_{-1}<x_0<y_{-1}<y_0<A, 0<x_{-1}<x_0<y_0<y_{-1}<A, 0<x_{-1}<y_{-1}<x_0<y_0<A,$$
$$0<x_{-1}<y_{-1}<y_0<x_0<A, 0<x_{-1}<y_0<y_{-1}<x_0<A, 0<x_{-1}<y_0<x_0<y_{-1}<A,$$
$$0<y_0<x_0<y_{-1}<x_{-1}<A, 0<y_0<x_0<x_{-1}<y_{-1}<A, 0<y_0<y_{-1}<x_0<x_{-1}<A,$$
$$0<y_0<y_{-1}<x_{-1}<x_0<A, 0<y_0<x_{-1}<y_{-1}<x_0<A, 0<y_0<x_{-1}<x_0<y_{-1}<A,$$
$$0<y_{-1}<x_0<y_0<x_{-1}<A, 0<y_{-1}<y_0<x_0<x_{-1}<A, 0<y_{-1}<y_0<x_{-1}<x_0<A,$$
$$0<y_{-1}<x_{-1}<y_0<x_0<A, 0<y_{-1}<x_0<x_{-1}<y_0<A, 0<y_{-1}<x_{-1}<x_0<y_0<A,$$
$$0<x_0<y_{-1}<y_0<x_{-1}<A, 0<x_0<y_0<x_{-1}<y_{-1}<A, 0<x_0<y_0<y_{-1}<x_{-1}<A,$$
$$0<x_0<x_{-1}<y_0<y_{-1}<A, 0<x_0<x_{-1}<y_{-1}<y_0<A, 0<x_0<y_{-1}<x_{-1}<y_0<A$$

başlangıç şartlarına göre aşağıdaki şekilde ve 10 periyotludur:

$$x(n) = \left\{\frac{1}{x_{-1}}; \frac{1}{x_0 y_{-1}}; \frac{1}{y_0}; y_{-1}; y_0; \frac{1}{y_{-1}}; \frac{1}{y_0 x_{-1}}; \frac{1}{x_0}; x_{-1}; x_0; \quad \ldots\right\}$$
$$y(n) = \left\{\frac{1}{y_{-1}}; \frac{1}{y_0 x_{-1}}; \frac{1}{x_0}; x_{-1}; x_0; \frac{1}{x_{-1}}; \frac{1}{x_0 y_{-1}}; \frac{1}{y_0}; y_{-1}; y_0; \quad \ldots\right\}.$$

İspat:

Bu teoremin ispatını n nin değerleri için gösterelim.

$$x_1 = \max\left\{\frac{1}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \frac{1}{x_{-1}} \qquad y_1 = \max\left\{\frac{1}{y_{-1}}, \frac{x_0}{y_{-1}}\right\} = \frac{1}{y_{-1}}$$

$$x_2 = \max\left\{\frac{1}{x_0}, \frac{y_1}{x_0}\right\} = \max\left\{\frac{1}{x_0}, \frac{1}{x_0 y_{-1}}\right\} = \frac{1}{x_0 y_{-1}} \qquad y_2 = \max\left\{\frac{1}{y_0}, \frac{x_1}{y_0}\right\} = \max\left\{\frac{1}{y_0}, \frac{1}{y_0 x_{-1}}\right\} = \frac{1}{y_0 x_{-1}}$$

$$x_3 = \max\left\{\frac{1}{x_1}, \frac{y_2}{x_1}\right\} = \max\left\{x_{-1}, \frac{1}{y_0}\right\} = \frac{1}{y_0} \qquad y_3 = \max\left\{\frac{1}{y_1}, \frac{x_2}{y_1}\right\} = \max\left\{y_{-1}, \frac{1}{x_0}\right\} = \frac{1}{x_0}$$

$$x_4 = \max\left\{\frac{1}{x_2}, \frac{y_3}{x_2}\right\} = \max\left\{x_0 y_{-1}, y_{-1}\right\} = y_{-1} \qquad y_4 = \max\left\{\frac{1}{y_2}, \frac{x_3}{y_2}\right\} = \max\left\{y_0 x_{-1}, x_{-1}\right\} = x_{-1}$$

$$x_5 = \max\left\{\frac{1}{x_3}, \frac{y_4}{x_3}\right\} = \max\left\{y_0, y_0 x_{-1}\right\} = y_0 \qquad y_5 = \max\left\{\frac{1}{y_3}, \frac{x_4}{y_3}\right\} = \max\left\{x_0, x_0 y_{-1}\right\} = x_0$$

$$x_6 = \max\left\{\frac{1}{x_4}, \frac{y_5}{x_4}\right\} = \max\left\{\frac{1}{y_{-1}}, \frac{x_0}{y_{-1}}\right\} = \frac{1}{y_{-1}} \qquad y_6 = \max\left\{\frac{1}{y_4}, \frac{x_5}{y_4}\right\} = \max\left\{\frac{1}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \frac{1}{x_{-1}}$$

$$x_7 = \max\left\{\frac{1}{x_5}, \frac{y_6}{x_5}\right\} = \max\left\{\frac{1}{y_0}, \frac{1}{y_0 x_{-1}}\right\} = \frac{1}{y_0 x_{-1}} \qquad y_7 = \max\left\{\frac{1}{y_5}, \frac{x_6}{y_5}\right\} = \max\left\{\frac{1}{x_0}, \frac{1}{x_0 y_{-1}}\right\} = \frac{1}{x_0 y_{-1}}$$

$$x_8 = \max\left\{\frac{1}{x_6}, \frac{y_7}{x_6}\right\} = \max\left\{y_{-1}, \frac{1}{x_0}\right\} = \frac{1}{x_0} \qquad y_8 = \max\left\{\frac{1}{y_6}, \frac{x_7}{y_6}\right\} = \max\left\{x_{-1}, \frac{1}{y_0}\right\} = \frac{1}{y_0}$$

$$x_9 = \max\left\{\frac{1}{x_7}, \frac{y_8}{x_7}\right\} = \max\left\{x_{-1} y_0, x_{-1}\right\} = x_{-1} \qquad y_9 = \max\left\{\frac{1}{y_7}, \frac{x_8}{y_7}\right\} = \max\left\{x_0 y_{-1}, y_{-1}\right\} = y_{-1}$$

$$x_{10} = \max\left\{\frac{1}{x_8}, \frac{y_9}{x_8}\right\} = \max\left\{x_0, x_0 y_{-1}\right\} = x_0 \qquad y_{10} = \max\left\{\frac{1}{y_8}, \frac{x_9}{y_8}\right\} = \max\left\{y_0, x_{-1} y_0\right\} = y_0$$

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elde edilir. Böylece

$$x(n) = \left\{\frac{1}{x_{-1}}; \frac{1}{x_0 y_{-1}}; \frac{1}{y_0}; y_{-1}; y_0; \frac{1}{y_{-1}}; \frac{1}{y_0 x_{-1}}; \frac{1}{x_0}; x_{-1}; x_0; \quad \ldots\right\}$$

$$y(n) = \left\{\frac{1}{y_{-1}}; \frac{1}{y_0 x_{-1}}; \frac{1}{x_0}; x_{-1}; x_0; \frac{1}{x_{-1}}; \frac{1}{x_0 y_{-1}}; \frac{1}{y_0}; y_{-1}; y_0; \quad \ldots\right\}$$

çözümlerinin 10 periyotlu olduğu gösterilmiş olur.

Teorem 2: Eğer A<1 ise $x_{n+1}=\max\left\{\frac{A}{x_{n-1}},\frac{y_n}{x_{n-1}}\right\};y_{n+1}=\max\left\{\frac{A}{y_{n-1}},\frac{x_n}{y_{n-1}}\right\}$

denkleminin $(x_n;y_n)$ çözümleri

$A<x_0<y_{-1}<y_0<x_{-1}<1, A<x_0<y_0<x_{-1}<y_{-1}<1, A<x_0<y_0<y_{-1}<x_{-1}<1,$

$A<y_0<x_{-1}<x_0<y_{-1}<1,\ A<y_0<x_0<y_{-1}<x_{-1}<1, A<x_{-1}<x_0<y_{-1}<y_0<1,$

$A<x_{-1}<x_0<y_0<y_{-1}<1, A<x_{-1}<y_0<x_0<y_{-1}<1, A<x_0<x_{-1}<y_{-1}<y_0<1,$

$A<x_0<x_{-1}<y_0<y_{-1}<1, A<x_0<y_{-1}<x_{-1}<y_0<1, A<x_{-1}<y_{-1}<x_0<y_0<1,$

$A<x_{-1}<y_{-1}<y_0<x_0<1, A<x_{-1}<y_0<y_{-1}<x_0<1, A<y_{-1}<x_{-1}<x_0<y_0<1,$

$A<y_{-1}<x_{-1}<y_0<x_0<1, A<y_{-1}<x_0<x_{-1}<y_0<1, A<y_0<x_{-1}<y_{-1}<x_0<1,$

$A<y_0<y_{-1}<x_{-1}<x_0<1, A<y_0<y_{-1}<x_0<x_{-1}<1, A<y_{-1}<x_0<y_0<x_{-1}<1,$

$A<y_{-1}<y_0<x_{-1}<x_0<1, A<y_{-1}<y_0<x_0<x_{-1}<1, A<y_0<x_0<x_{-1}<y_{-1}<1,$

başlangıç şartlarına göre aşağıdaki şekilde ve 6 periyotludur:

$$x(n)=\left\{\frac{y_0}{x_{-1}};\frac{1}{y_{-1}};\frac{1}{y_0};\frac{y_{-1}}{x_0};x_{-1};x_0;\quad\ldots\right\}$$
$$y(n)=\left\{\frac{x_0}{y_{-1}};\frac{1}{x_{-1}};\frac{1}{x_0};\frac{x_{-1}}{y_0};y_{-1};y_0;\quad\ldots\right\}.$$

İspat:

Bu teoremin ispatını n nin değerleri için gösterelim.

$x_1=\max\left\{\frac{A}{x_{-1}},\frac{y_0}{x_{-1}}\right\}=\frac{y_0}{x_{-1}}$ $\qquad$ $y_1=\max\left\{\frac{A}{y_{-1}},\frac{x_0}{y_{-1}}\right\}=\frac{y_0}{x_{-1}}$

$x_2=\max\left\{\frac{A}{x_0},\frac{y_1}{x_0}\right\}=\max\left\{\frac{A}{x_0},\frac{1}{y_{-1}}\right\}=\frac{1}{y_{-1}}$ $\qquad$ $y_2=\max\left\{\frac{A}{y_0},\frac{x_1}{y_0}\right\}=\max\left\{\frac{A}{y_0},\frac{1}{x_{-1}}\right\}=\frac{1}{x_{-1}}$

$x_3=\max\left\{\frac{A}{x_1},\frac{y_2}{x_1}\right\}=\max\left\{\frac{Ax_{-1}}{y_0},\frac{1}{y_0}\right\}=\frac{1}{y_0}$ $\qquad$ $y_3=\max\left\{\frac{1}{y_1},\frac{x_2}{y_1}\right\}=\max\left\{\frac{Ay_{-1}}{x_0},\frac{1}{x_0}\right\}=\frac{1}{x_0}$

$x_4=\max\left\{\frac{1}{x_2},\frac{y_3}{x_2}\right\}=\max\left\{A\,y_{-1},\frac{y_{-1}}{x_0}\right\}=\frac{y_{-1}}{x_0}$ $\qquad$ $y_4=\max\left\{\frac{A}{y_2},\frac{x_3}{y_2}\right\}=\max\left\{A\,x_{-1},\frac{x_{-1}}{y_0}\right\}=\frac{x_{-1}}{y_0}$

$x_5=\max\left\{\frac{A}{x_3},\frac{y_4}{x_3}\right\}=\max\left\{A\,y_0,x_{-1}\right\}=x_{-1}$ $\qquad$ $y_5=\max\left\{\frac{A}{y_3},\frac{x_4}{y_3}\right\}=\max\left\{A\,x_0,y_{-1}\right\}=y_{-1}$

$$x_6=\max\left\{\frac{A}{x_4},\frac{y_5}{x_4}\right\}=\max\left\{\frac{Ax_0}{y_{-1}},x_0\right\}=x_0 \qquad y_6=\max\left\{\frac{A}{y_4},\frac{x_5}{y_4}\right\}=\max\left\{\frac{Ay_0}{x_{-1}},y_0\right\}=y_0$$

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elde edilir. Böylece

$$x(n)=\left\{\frac{y_0}{x_{-1}};\frac{1}{y_{-1}};\frac{1}{y_0};\frac{y_{-1}}{x_0};x_{-1};x_0;\quad\dots\right\}$$
$$y(n)=\left\{\frac{x_0}{y_{-1}};\frac{1}{x_{-1}};\frac{1}{x_0};\frac{x_{-1}}{y_0};y_{-1};y_0;\quad\dots\right\}$$

çözümlerinin 6 periyotlu olduğu gösterilmiş olur.

Teorem 3: Eğer $x_{-1},x_0,y_{-1},y_0,A>1$ ise

$x_{n+1}=\max\left\{\frac{A}{x_{n-1}},\frac{y_n}{x_{n-1}}\right\};y_{n+1}=\max\left\{\frac{A}{y_{n-1}},\frac{x_n}{y_{n-1}}\right\}$ denkleminin $(x_n;y_n)$ çözümleri

$$x_{-1}<x_0<y_{-1}<y_0<A, x_{-1}<x_0<y_0<y_{-1}<A, x_{-1}<y_{-1}<x_0<y_0<A,$$
$$x_{-1}<y_{-1}<y_0<x_0<A, x_{-1}<y_0<y_{-1}<x_0<A, x_{-1}<y_0<x_0<y_{-1}<A,$$
$$y_0<x_0<y_{-1}<x_{-1}<A, y_0<x_0<x_{-1}<y_{-1}<A, y_0<y_{-1}<x_0<x_{-1}<A,$$
$$y_0<y_{-1}<x_{-1}<x_0<A, y_0<x_{-1}<y_{-1}<x_0<A, y_0<x_{-1}<x_0<y_{-1}<A,$$
$$y_{-1}<x_0<y_0<x_{-1}<A, y_{-1}<y_0<x_0<x_{-1}<A, y_{-1}<y_0<x_{-1}<x_0<A,$$
$$y_{-1}<x_{-1}<y_0<x_0<A, y_{-1}<x_0<x_{-1}<y_0<A, y_{-1}<x_{-1}<x_0<y_0<A,$$
$$x_0<y_{-1}<y_0<x_{-1}<A, x_0<y_0<x_{-1}<y_{-1}<A, x_0<y_0<y_{-1}<x_{-1}<A,$$
$$x_0<x_{-1}<y_0<y_{-1}<A, x_0<x_{-1}<y_{-1}<y_0<A, x_0<y_{-1}<x_{-1}<y_0<A$$

başlangıç şartlarına göre aşağıdaki şekilde ve 4 periyotludur:

$$x(n)=\left\{\frac{A}{x_{-1}};\frac{A}{x_0};x_{-1};x_0\dots\right\}$$
$$y(n)=\left\{\frac{A}{y_{-1}};\frac{A}{y_0};y_{-1};y_0\dots\right\}.$$

İspat:

Bu teoremin ispatını n nin değerleri için gösterelim.

$$x_1=\max\left\{\frac{A}{x_{-1}},\frac{y_0}{x_{-1}}\right\}=\frac{A}{x_{-1}} \qquad y_1=\max\left\{\frac{A}{y_{-1}},\frac{x_0}{y_{-1}}\right\}=\frac{A}{y_{-1}}$$

$$x_2=\max\left\{\frac{A}{x_0},\frac{y_1}{x_0}\right\}=\max\left\{\frac{A}{x_0},\frac{A}{x_0y_{-1}}\right\}=\frac{A}{x_0} \qquad y_2=\max\left\{\frac{A}{y_0},\frac{x_1}{y_0}\right\}=\max\left\{\frac{A}{y_0},\frac{A}{y_0x_{-1}}\right\}=\frac{A}{y_0}$$

$$x_3=\max\left\{\frac{A}{x_1},\frac{y_2}{x_1}\right\}=\max\left\{x_{-1},\frac{x_{-1}}{Ay_0}\right\}=x_{-1} \qquad y_3=\max\left\{\frac{A}{y_1},\frac{x_2}{y_1}\right\}=\max\left\{y_{-1},\frac{y_{-1}}{x_0}\right\}=y_{-1}$$

$$x_4=\max\left\{\frac{A}{x_2},\frac{y_3}{x_2}\right\}=\max\left\{x_0,\frac{x_0y_{-1}}{A}\right\}=x_0 \qquad y_4=\max\left\{\frac{A}{y_2},\frac{x_3}{y_2}\right\}=\max\left\{y_0,\frac{x_{-1}y_0}{A}\right\}=y_0$$

$$x_5=\max\left\{\frac{A}{x_3},\frac{y_4}{x_3}\right\}=\max\left\{\frac{A}{x_{-1}},\frac{y_0}{x_{-1}}\right\}=\frac{A}{x_{-1}} \qquad y_5=\max\left\{\frac{A}{y_3},\frac{x_4}{y_3}\right\}=\max\left\{\frac{A}{y_{-1}},\frac{x_0}{y_{-1}}\right\}=\frac{A}{y_{-1}}$$

$$x_6=\max\left\{\frac{A}{x_4},\frac{y_5}{x_4}\right\}=\max\left\{\frac{A}{x_0},\frac{A}{x_0y_{-1}}\right\}=\frac{A}{x_0} \qquad y_6=\max\left\{\frac{A}{y_4},\frac{x_5}{y_4}\right\}=\max\left\{\frac{A}{y_0},\frac{A}{x_{-1}y_0}\right\}=\frac{A}{y_0}$$

$$x_7=\max\left\{\frac{A}{x_5},\frac{y_6}{x_5}\right\}=\max\left\{x_{-1},\frac{x_{-1}}{y_0}\right\}=x_{-1} \qquad y_7=\max\left\{\frac{A}{y_5},\frac{x_6}{y_5}\right\}=\max\left\{y_{-1},\frac{y_{-1}}{x_0}\right\}=y_{-1}$$

$$x_8=\max\left\{\frac{A}{x_6},\frac{y_7}{x_6}\right\}=\max\left\{x_0,\frac{x_0y_{-1}}{A}\right\}=x_0 \qquad y_8=\max\left\{\frac{A}{y_6},\frac{x_7}{y_6}\right\}=\max\left\{y_0,\frac{x_{-1}y_0}{A}\right\}=y_0$$

$$x_9=\max\left\{\frac{A}{x_7},\frac{y_8}{x_7}\right\}=\max\left\{\frac{A}{x_{-1}},\frac{y_0}{x_{-1}}\right\}=\frac{A}{x_{-1}} \qquad y_9=\max\left\{\frac{A}{y_7},\frac{x_8}{y_7}\right\}=\max\left\{\frac{A}{y_{-1}},\frac{x_0}{y_{-1}}\right\}=\frac{A}{y_{-1}}$$

$$x_{10}=\max\left\{\frac{A}{x_8},\frac{y_9}{x_8}\right\}=\max\left\{\frac{A}{x_0},\frac{A}{x_0y_{-1}}\right\}=\frac{A}{x_0} \qquad y_{10}=\max\left\{\frac{A}{y_8},\frac{x_9}{y_8}\right\}=\max\left\{\frac{A}{y_0},\frac{A}{x_{-1}y_0}\right\}=\frac{A}{y_0}$$

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elde edilir. Böylece

$$x(n)=\left\{\frac{A}{x_{-1}};\frac{A}{x_0};x_{-1};x_0\ldots\right\}$$

$$y(n)=\left\{\frac{A}{y_{-1}};\frac{A}{y_0};y_{-1};y_0\ldots\right\}$$

çözümlerinin 4 periyotlu olduğu gösterilmiş olur.

Teorem 4.1: Eğer A<1 ise $x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$

denkleminin $(x_n; y_n)$ çözümleri $x_0 < x_{-1} < y_0 < y_{-1} < A < 1$,

$y_0 < x_{-1} < x_0 < y_{-1} < A < 1$ başlangıç şartlarına göre aşağıdaki şekildedir. n=0,1,2,... için

X_n ÇÖZÜMLERİ

$$x_{10n+1} = \frac{A^{2n+1}}{x_{-1}};$$

n=0,1 için $x_{10n+2} = \frac{A}{x_0 y_{-1}}$, $n \geq 2$ için $x_{10n+2} = \frac{A^{2n+2}}{x_0 y_{-1}^2}$;

n=0,1,2,3,4,5 için $x_{10n+3} = \frac{1}{A^{2n} y_0}$, $n \geq 6$ için $x_{10n+3} = \frac{A}{x_{-1} y_0}$;

$$x_{10n+4} = \frac{y_{-1}}{A^{2n+1}};$$

n=0,1,2,3,4 için $x_{10n+5} = A^{2n+1} \cdot y_0$, n=5 için $x_{10n+5} = \frac{x_{-1} y_0}{A}$, $n \geq 6$ için

$$x_{10n+5} = \frac{x_{-1}^2 y_0}{A^{2n+2}};$$

$$x_{10n+6} = \frac{A^{2n+2}}{y_{-1}};$$

n=0,1,2,3,4 için $x_{10n+7} = \frac{A}{x_{-1} y_0}$, $n \geq 5$ $x_{10n+7} = \frac{A^{2n+3}}{x_{-1}^2 y_0}$;

n=0 için $x_{10n+8} = \frac{1}{A x_0}$, $n \geq 1$ için $x_{10n+8} = \frac{A}{x_0 y_{-1}}$;

$$x_{10n+9} = \frac{x_{-1}}{A^{2n+2}};$$

n=0 için $x_{10n+10} = A^2 \cdot x_0$, $n \geq 1$ için $x_{10n+10} = \frac{x_0 y_{-1}^2}{A^{2n+3}}$;

Y_n ÇÖZÜMLERİ

$$y_{10n+1}=\frac{A^{2n+1}}{y_{-1}};$$

n=0,1,2,3,4,5 için $y_{10n+2}=\frac{A}{x_{-1}y_0}$, $n\geq 6$ için $y_{10n+2}=\frac{A^{2n+2}}{x_{-1}^2 y_0}$;

n=0,1 için $y_{10n+3}=\frac{1}{A^{2n}x_0}$, $n\geq 2$ için $y_{10n+3}=\frac{A}{x_0 y_{-1}}$;

$$y_{10n+4}=\frac{x_{-1}}{A^{2n+1}};$$

n=0 için $y_{10n+5}=Ax_0$, $n=1$ için $y_{10n+5}=\frac{x_0 y_{-1}}{A}$, $n\geq 2$ için $y_{10n+5}=\frac{x_0 y_{-1}^2}{A^{2n+2}}$;

$$y_{10n+6}=\frac{A^{2n+2}}{x_{-1}};$$

n=0 için $y_{10n+7}=\frac{A}{x_0 y_{-1}}$, $n\geq 1$ için $y_{10n+7}=\frac{A^{2n+3}}{x_0 y_{-1}^2}$;

n=0,1,2,3,4,5 için $y_{10n+8}=\frac{1}{A^{2n+1}y_0}$, $n\geq 6$ $y_{10n+8}=\frac{A}{x_{-1}y_0}$;

$$y_{10n+9}=\frac{y_{-1}}{A^{2n+2}};$$

n=0,1,2,3,4 için $y_{10n+10}=A^{2n+2}\cdot y_0$, $n\geq 5$ $y_{10n+10}=\frac{x_{-1}^2 y_0}{A^{2n+3}}$;

İspat:

Bu teoremin ispatını n nin değerleri için gösterelim.

$$x_1=\max\left\{\frac{A}{x_{-1}},\frac{y_0}{x_{-1}}\right\}=\frac{A}{x_{-1}} \qquad y_1=\max\left\{\frac{A}{y_{-1}},\frac{x_0}{y_{-1}}\right\}=\frac{A}{y_{-1}}$$

$$x_2=\max\left\{\frac{A}{x_0},\frac{y_1}{x_0}\right\}=\max\left\{\frac{A}{x_0},\frac{A}{y_{-1}x_0}\right\}=\frac{A}{y_{-1}x_0} \qquad y_2=\max\left\{\frac{A}{y_0},\frac{x_1}{y_0}\right\}=\max\left\{\frac{A}{y_0},\frac{A}{x_{-1}y_0}\right\}=\frac{A}{x_{-1}y_0}$$

$$x_3=\max\left\{\frac{A}{x_1},\frac{y_2}{x_1}\right\}=\max\left\{x_{-1},\frac{1}{y_0}\right\}=\frac{1}{y_0} \qquad y_3=\max\left\{\frac{A}{y_1},\frac{x_2}{y_1}\right\}=\max\left\{\frac{Ay_{-1}}{A},\frac{Ay_{-1}}{x_0Ay_{-1}}\right\}=\frac{1}{x_0}$$

$$x_4=\max\left\{\frac{A}{x_2},\frac{y_3}{x_2}\right\}=\max\left\{x_0y_{-1},\frac{y_{-1}}{A}\right\}=\frac{y_{-1}}{A} \qquad y_4=\max\left\{\frac{A}{y_2},\frac{x_3}{y_2}\right\}=\max\left\{y_0x_{-1},\frac{x_{-1}}{A}\right\}=\frac{x_{-1}}{A}$$

$$x_5 = \max\left\{\frac{A}{x_3}, \frac{y_4}{x_3}\right\} = \max\left\{\mathrm{A}\, y_0, x_{-1}\frac{y_0}{A}\right\} = \mathrm{A}\, y_0 \qquad y_5 = \max\left\{\frac{A}{y_3}, \frac{x_4}{y_3}\right\} = \max\left\{\mathrm{A}\, x_0, y_{-1}\frac{x_0}{A}\right\} = \mathrm{A}\, x_0$$

$$x_6 = \max\left\{\frac{A}{x_4}, \frac{y_5}{x_4}\right\} = \max\left\{\frac{A^2}{y_{-1}}, \frac{A^2 x_0}{y_{-1}}\right\} = \frac{A^2}{y_{-1}} \qquad y_6 = \max\left\{\frac{A}{y_4}, \frac{x_5}{y_4}\right\} = \max\left\{\frac{A^2}{x_{-1}}, \frac{A^2 y_0}{x_{-1}}\right\} = \frac{A^2}{x_{-1}}$$

$$x_7 = \max\left\{\frac{A}{x_5}, \frac{y_6}{x_5}\right\} = \left\{\frac{1}{y_0}, \frac{A}{x_{-1}y_0}\right\} = \frac{A}{x_{-1}y_0} \qquad y_7 = \max\left\{\frac{A}{y_5}, \frac{x_6}{y_5}\right\} = \left\{\frac{1}{x_0}, \frac{A}{x_0 y_{-1}}\right\} = \frac{A}{x_0 y_{-1}}$$

$$x_8 = \max\left\{\frac{A}{x_6}, \frac{y_7}{x_6}\right\} = \max\left\{\frac{y_{-1}}{A}, \frac{1}{Ax_0}\right\} = \frac{1}{Ax_0} \qquad y_8 = \max\left\{\frac{A}{y_6}, \frac{x_7}{y_6}\right\} = \max\left\{\frac{x_{-1}}{A}, \frac{1}{Ay_0}\right\} = \frac{1}{Ay_0}$$

$$x_9 = \max\left\{\frac{A}{x_7}, \frac{y_8}{x_7}\right\} = \max\left\{\mathrm{y}_0 \mathrm{x}_{-1}, \frac{x_{-1}}{A^2}\right\} = \frac{x_{-1}}{A^2} \qquad y_9 = \max\left\{\frac{A}{y_7}, \frac{x_8}{y_7}\right\} = \left\{\mathrm{x}_0 \mathrm{y}_{-1}, \frac{y_{-1}}{A^2}\right\} = \frac{y_{-1}}{A^2}$$

$$x_{10} = \max\left\{\frac{A}{x_8}, \frac{y_9}{x_8}\right\} = \max\left\{A^2 x_0, \frac{Ax_0 y_{-1}}{A^2}\right\} = A^2 x_0 \qquad y_{10} = \max\left\{\frac{A}{y_8}, \frac{x_9}{y_8}\right\} = \max\left\{A^2 y_0, \frac{Ay_0 x_{-1}}{A^2}\right\} = A^2 y_0$$

$$x_{11} = \max\left\{\frac{A}{x_9}, \frac{y_{10}}{x_9}\right\} = \max\left\{\frac{A^3}{x_{-1}}, \frac{A^4 y_0}{x_{-1}}\right\} = \frac{A^3}{x_{-1}} \qquad y_{11} = \max\left\{\frac{A}{y_9}, \frac{x_{10}}{y_9}\right\} = \max\left\{\frac{A^3}{y_{-1}}, \frac{A^4 x_0}{y_{-1}}\right\} = \frac{A^3}{y_{-1}}$$

$$x_{12} = \max\left\{\frac{A}{x_{10}}, \frac{y_{11}}{x_{10}}\right\} = \max\left\{\frac{1}{Ax_0}, \frac{A}{x_0 y_{-1}}\right\} = \frac{A}{x_0 y_{-1}} \qquad y_{12} = \max\left\{\frac{A}{y_{10}}, \frac{x_{11}}{y_{10}}\right\} = \max\left\{\frac{1}{Ay_0}, \frac{A}{y_0 x_{-1}}\right\} = \frac{A}{y_0 x_{-1}}$$

$$x_{13} = \max\left\{\frac{A}{x_{11}}, \frac{y_{12}}{x_{11}}\right\} = \left\{\frac{x_{-1}}{A^2}, \frac{1}{A^2 y_0}\right\} = \frac{1}{A^2 y_0} \qquad y_{13} = \max\left\{\frac{A}{y_{11}}, \frac{x_{12}}{y_{11}}\right\} = \left\{\frac{y_{-1}}{A^2}, \frac{1}{A^2 x_0}\right\} = \frac{1}{A^2 x_0}$$

$$x_{14} = \max\left\{\frac{A}{x_{12}}, \frac{y_{13}}{x_{12}}\right\} = \max\left\{\mathrm{x}_0 \mathrm{y}_{-1}, \frac{y_{-1}}{A^3}\right\} = \frac{y_{-1}}{A^3} \qquad y_{14} = \max\left\{\frac{A}{y_{12}}, \frac{x_{13}}{y_{12}}\right\} = \max\left\{\mathrm{y}_0 \mathrm{x}_{-1}, \frac{x_{-1}}{A^3}\right\} = \frac{x_{-1}}{A^3}$$

$$x_{15} = \max\left\{\frac{A}{x_{13}}, \frac{y_{14}}{x_{13}}\right\} = \max\left\{A^3\, \mathrm{y}_0, \frac{y_0 x_{-1}}{A}\right\} = A^3\, \mathrm{y}_0 \qquad y_{15} = \max\left\{\frac{A}{y_{13}}, \frac{x_{14}}{y_{13}}\right\} = \max\left\{\mathrm{A}^3 \mathrm{x}_0, \frac{x_0 y_{-1}}{A}\right\} = \frac{x_0 y_{-1}}{A}$$

$$x_{16} = \max\left\{\frac{A}{x_{14}}, \frac{y_{15}}{x_{14}}\right\} = \max\left\{\frac{A^4}{y_{-1}}, \mathrm{x}_0\, A^2\right\} = \frac{A^4}{y_{-1}} \qquad y_{16} = \max\left\{\frac{A}{y_{14}}, \frac{x_{15}}{y_{14}}\right\} = \max\left\{\frac{A^4}{x_{-1}}, \frac{A^6 y_0}{x_{-1}}\right\} = \frac{A^4}{x_{-1}}$$

$$x_{17} = \max\left\{\frac{A}{x_{15}}, \frac{y_{16}}{x_{15}}\right\} = \max\left\{\frac{1}{A^2 y_0}, \frac{A}{y_0 x_{-1}}\right\} = \frac{A}{y_0 x_{-1}} \qquad y_{17} = \max\left\{\frac{A}{y_{15}}, \frac{x_{16}}{y_{15}}\right\} = \max\left\{\frac{\mathrm{A}^2}{x_0 y_{-1}}, \frac{A^5}{x_0 y_{-1}^{\;2}}\right\} = \frac{A^5}{x_0 y_{-1}^{\;2}}$$

$$x_{18} = \max\left\{\frac{A}{x_{16}}, \frac{y_{17}}{x_{16}}\right\} = \max\left\{\frac{y_{-1}}{A^3}, \frac{A}{x_0 y_{-1}}\right\} = \frac{A}{x_0 y_{-1}} \qquad y_{18} = \max\left\{\frac{A}{y_{16}}, \frac{x_5}{y_4}\right\} = \max\left\{\frac{x_{-1}}{A^3}, \frac{1}{y_0 A^3}\right\} = \frac{1}{y_0 A^3}$$

$$x_{19} = \max\left\{\frac{A}{x_{15}}, \frac{y_{16}}{x_{15}}\right\} = \max\left\{y_0 x_{-1}, \frac{x_{-1}}{A^4}\right\} = \frac{x_{-1}}{A^4} \qquad y_{19} = \max\left\{\frac{A}{y_{17}}, \frac{x_{18}}{y_{17}}\right\} = \max\left\{\frac{x_0 y_{-1}^{\;2}}{A^5}, \frac{y_{-1}}{A^4}\right\} = \frac{y_{-1}}{A^4}$$

$$x_{20} = \max\left\{\frac{A}{x_{16}}, \frac{y_{17}}{x_{16}}\right\} = \max\left\{x_0 y_{-1}, \frac{x_0 y_{-1}^{\;2}}{A^5}\right\} = \frac{x_0 y_{-1}^{\;2}}{A^5} \qquad y_{20} = \max\left\{\frac{A}{y_{16}}, \frac{x_5}{y_4}\right\} = \max\left\{A^4 y_0, \frac{x_{-1} y_0}{A}\right\} = A^4 y_0$$

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elde edilir.

Teorem 4.2: Eğer A<1 ise $x_{n+1}=\max\left\{\frac{A}{x_{n-1}},\frac{y_n}{x_{n-1}}\right\}; y_{n+1}=\max\left\{\frac{A}{y_{n-1}},\frac{x_n}{y_{n-1}}\right\}$ denkleminin $(x_n; y_n)$ çözümleri $x_0<x_{-1}<y_{-1}<y_0<A<1, y_0<x_{-1}<y_{-1}<x_0<A<1$ başlangıç şartlarına göre aşağıdaki şekildedir:

X_n ÇÖZÜMLERİ

$$x_{10n+1}=\frac{A^{2n+1}}{x_{-1}};$$

n=0,1,2 için $x_{10n+2}=\frac{A}{x_0y_{-1}}$, $n\geq 3$ için $x_{10n+2}=\frac{A^{2n+2}}{x_0y_{-1}^2}$;

n=0,1,2,3,4,5 için $x_{10n+3}=\frac{1}{A^{2n}y_0}$, $n\geq 6$ için $x_{10n+3}=\frac{A}{x_{-1}y_0}$;

$$x_{10n+4}=\frac{y_{-1}}{A^{2n+1}};$$

n=0,1,2,3,4 için $x_{10n+5}=A^{2n+1}\cdot y_0$, n=5 için $x_{10n+5}=\frac{x_{-1}y_0}{A}$, $n\geq 6$ için

$$x_{10n+5}=\frac{x_{-1}^2y_0}{A^{2n+2}};$$

$$x_{10n+6}=\frac{A^{2n+2}}{y_{-1}};$$

n=0,1,2,3,4 için $x_{10n+7}=\frac{A}{x_{-1}y_0}$, $n\geq 5$ için $x_{10n+7}=\frac{A^{2n+3}}{x_{-1}^2y_0}$;

n=0,1,2 için $x_{10n+8}=\frac{1}{A^{2n+1}x_0}$, $n\geq 3$ için $x_{10n+8}=\frac{A}{x_0y_{-1}}$;

$$x_{10n+9}=\frac{x_{-1}}{A^{2n+2}};$$

n=0,1 için $x_{10n+10} = A^{2n+2} \cdot x_0$, $n = 2$ için $x_{10n+10} = \dfrac{x_0 y_{-1}}{A}$, $n \geq 3$ için

$$x_{10n+10} = \frac{x_0 y_{-1}^2}{A^{2n+3}};$$

Y_n ÇÖZÜMLERİ

$$y_{10n+1} = \frac{A^{2n+1}}{y_{-1}};$$

n=0,1,2,3,4,5 için $y_{10n+2} = \dfrac{A}{x_{-1} y_0}$, $n \geq 6$ için $y_{10n+2} = \dfrac{A^{2n+2}}{x_{-1}^2 y_0}$;

n=0,1,2 için $y_{10n+3} = \dfrac{1}{A^{2n} x_0}$, $n \geq 3$ için $y_{10n+3} = \dfrac{A}{x_0 y_{-1}}$;

$$y_{10n+4} = \frac{x_{-1}}{A^{2n+1}};$$

n=0,1,2 için $y_{10n+5} = A^{2n+1} \cdot x_0$, $n \geq 3$ iç in $y_{10n+5} = \dfrac{x_0 y_{-1}^2}{A^{2n+2}}$;

$$y_{10n+6} = \frac{A^{2n+2}}{x_{-1}};$$

n=0,1,2 için $y_{10n+7} = \dfrac{A}{x_0 y_{-1}}$, $n \geq 3$ için $y_{10n+7} = \dfrac{A^{2n+3}}{x_0 y_{-1}^2}$;

n=0,1,2,3,4 için $y_{10n+8} = \dfrac{1}{A^{2n+1} y_0}$, $n \geq 5$ için $y_{10n+8} = \dfrac{A}{x_{-1} y_0}$;

$$y_{10n+9} = \frac{y_{-1}}{A^{2n+2}};$$

n=0,1,2,3,4 için $y_{10n+10} = A^{2n+2} \cdot y_0$, $n \geq 5$ için $y_{10n+10} = \dfrac{x_{-1}^2 y_0}{A^{2n+3}}$;

İspat:

Bu teoremin ispatını n nin değerleri için gösterelim.

$$x_1 = \max\left\{\frac{A}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \frac{A}{x_{-1}} \qquad y_1 = \max\left\{\frac{A}{y_{-1}}, \frac{x_0}{y_{-1}}\right\} = \frac{A}{y_{-1}}$$

$$x_2 = \max\left\{\frac{A}{x_0}, \frac{y_1}{x_0}\right\} = \max\left\{\frac{A}{x_0}, \frac{A}{y_{-1} x_0}\right\} = \frac{A}{y_{-1} x_0} \qquad y_2 = \max\left\{\frac{A}{y_0}, \frac{x_1}{y_0}\right\} = \max\left\{\frac{A}{y_0}, \frac{A}{x_{-1} y_0}\right\} = \frac{A}{x_{-1} y_0}$$

$$x_3=\max\left\{\frac{A}{x_0},\frac{y_1}{x_0}\right\}=\max\left\{x_{-1},\frac{1}{y_0}\right\}=\frac{1}{y_0}\qquad y_3=\max\left\{\frac{A}{y_1},\frac{x_2}{y_1}\right\}=\max\left\{\frac{Ay_{-1}}{A},\frac{Ay_{-1}}{x_0Ay_{-1}}\right\}=\frac{1}{x_0}$$

$$x_4=\max\left\{\frac{A}{x_2},\frac{y_3}{x_2}\right\}=\max\left\{x_0y_{-1},\frac{y_{-1}}{A}\right\}=\frac{y_{-1}}{A}\qquad y_4=\max\left\{\frac{A}{y_2},\frac{x_3}{y_2}\right\}=\max\left\{y_0x_{-1},\frac{x_{-1}}{A}\right\}=\frac{x_{-1}}{A}$$

$$x_5=\max\left\{\frac{A}{x_3},\frac{y_4}{x_3}\right\}=\max\left\{A\,y_0,x_{-1}\frac{y_0}{A}\right\}=A\,y_0\qquad y_5=\max\left\{\frac{A}{y_3},\frac{x_4}{y_3}\right\}=\max\left\{A\,x_0,y_{-1}\frac{x_0}{A}\right\}=A\,x_0$$

$$x_6=\max\left\{\frac{A}{x_4},\frac{y_5}{x_4}\right\}=\max\left\{\frac{A^2}{y_{-1}},\frac{A^2x_0}{y_{-1}}\right\}=\frac{A^2}{y_{-1}}\qquad y_6=\max\left\{\frac{A}{y_4},\frac{x_5}{y_4}\right\}=\max\left\{\frac{A^2}{x_{-1}},\frac{A^2y_0}{x_{-1}}\right\}=\frac{A^2}{x_{-1}}$$

$$x_7=\max\left\{\frac{A}{x_{-1}},\frac{y_0}{x_{-1}}\right\}=\left\{\frac{1}{y_0},\frac{A}{x_{-1}y_0}\right\}=\frac{A}{x_{-1}y_0}\qquad y_7=\max\left\{\frac{A}{y_5},\frac{x_6}{y_5}\right\}=\left\{\frac{1}{x_0},\frac{A}{x_0y_{-1}}\right\}=\frac{A}{x_0y_{-1}}$$

$$x_8=\max\left\{\frac{A}{x_6},\frac{y_7}{x_6}\right\}=\max\left\{\frac{y_{-1}}{A},\frac{1}{Ax_0}\right\}=\frac{1}{Ax_0}\qquad y_8=\max\left\{\frac{A}{y_6},\frac{x_7}{y_6}\right\}=\max\left\{\frac{x_{-1}}{A},\frac{1}{Ay_0}\right\}=\frac{1}{Ay_0}$$

$$x_9=\max\left\{\frac{A}{x_7},\frac{y_8}{x_7}\right\}=\max\left\{y_0x_{-1},\frac{x_{-1}}{A^2}\right\}=\frac{x_{-1}}{A^2}\qquad y_9=\max\left\{\frac{A}{y_7},\frac{x_8}{y_7}\right\}=\left\{x_0y_{-1},\frac{y_{-1}}{A^2}\right\}=\frac{y_{-1}}{A^2}$$

$$x_{10}=\max\left\{\frac{A}{x_8},\frac{y_9}{x_8}\right\}=\max\left\{A^2x_0,\frac{Ax_0y_{-1}}{A^2}\right\}=A^2x_0\qquad y_{10}=\max\left\{\frac{A}{y_8},\frac{x_9}{y_8}\right\}=\max\left\{A^2y_0,\frac{Ay_0x_{-1}}{A^2}\right\}=A^2y_0$$

$$x_{11}=\max\left\{\frac{A}{x_9},\frac{y_{10}}{x_9}\right\}=\max\left\{\frac{A^3}{x_{-1}},\frac{A^4y_0}{x_{-1}}\right\}=\frac{A^3}{x_{-1}}\qquad y_{11}=\max\left\{\frac{A}{y_9},\frac{x_{10}}{y_9}\right\}=\max\left\{\frac{A^3}{y_{-1}},\frac{A^4x_0}{y_{-1}}\right\}=\frac{A^3}{y_{-1}}$$

$$x_{12}=\max\left\{\frac{A}{x_{10}},\frac{y_{11}}{x_{10}}\right\}=\max\left\{\frac{1}{Ax_0},\frac{A}{x_0y_{-1}}\right\}=\frac{A}{x_0y_{-1}}\qquad y_{12}=\max\left\{\frac{A}{y_{10}},\frac{x_{11}}{y_{10}}\right\}=\max\left\{\frac{1}{Ay_0},\frac{A}{y_0x_{-1}}\right\}=\frac{A}{y_0x_{-1}}$$

$$x_{13}=\max\left\{\frac{A}{x_{11}},\frac{y_{12}}{x_{11}}\right\}=\left\{\frac{x_{-1}}{A^2},\frac{1}{A^2y_0}\right\}=\frac{1}{A^2y_0}\qquad y_{13}=\max\left\{\frac{A}{y_{11}},\frac{x_{12}}{y_{11}}\right\}=\left\{\frac{y_{-1}}{A^2},\frac{1}{A^2x_0}\right\}=\frac{1}{A^2x_0}$$

$$x_{14}=\max\left\{\frac{A}{x_{12}},\frac{y_{13}}{x_{12}}\right\}=\max\left\{x_0y_{-1},\frac{y_{-1}}{A^3}\right\}=\frac{y_{-1}}{A^3}\qquad y_{14}=\max\left\{\frac{A}{y_{12}},\frac{x_{13}}{y_{12}}\right\}=\max\left\{y_0x_{-1},\frac{x_{-1}}{A^3}\right\}=\frac{x_{-1}}{A^3}$$

$$x_{15}=\max\left\{\frac{A}{x_{13}},\frac{y_{14}}{x_{13}}\right\}=\max\left\{A^3y_0,\frac{y_0x_{-1}}{A}\right\}=A^3y_0\qquad y_{15}=\max\left\{\frac{A}{y_{13}},\frac{x_{14}}{y_{13}}\right\}=\max\left\{A^3x_0,\frac{x_0y_{-1}}{A}\right\}=A^3x_0$$

$$x_{16}=\max\left\{\frac{A}{x_{14}},\frac{y_{15}}{x_{14}}\right\}=\max\left\{\frac{A^4}{y_{-1}},\frac{x_0A^6}{y_{-1}}\right\}=\frac{A^4}{y_{-1}}\qquad y_{16}=\max\left\{\frac{A}{y_{14}},\frac{x_{15}}{y_{14}}\right\}=\max\left\{\frac{A^4}{x_{-1}},\frac{A^6y_0}{x_{-1}}\right\}=\frac{A^4}{x_{-1}}$$

$$x_{17}=\max\left\{\frac{A}{x_{15}},\frac{y_{16}}{x_{15}}\right\}=\max\left\{\frac{1}{A^2y_0},\frac{A}{y_0x_{-1}}\right\}=\frac{A}{y_0x_{-1}}\qquad y_{17}=\max\left\{\frac{A}{y_{15}},\frac{x_{16}}{y_{15}}\right\}=\max\left\{\frac{1}{x_0A^2},\frac{A}{x_0y_{-1}}\right\}=\frac{A}{x_0y_{-1}}$$

$$x_{18}=\max\left\{\frac{A}{x_{16}},\frac{y_{17}}{x_{16}}\right\}=\max\left\{\frac{y_{-1}}{A^3},\frac{1}{x_0A^3}\right\}=\frac{1}{x_0A^3}\qquad y_{18}=\max\left\{\frac{A}{y_{16}},\frac{x_5}{y_4}\right\}=\max\left\{\frac{x_{-1}}{A^3},\frac{1}{y_0A^3}\right\}=\frac{1}{y_0A^3}$$

$$x_{19}=\max\left\{\frac{A}{x_{15}},\frac{y_{16}}{x_{15}}\right\}=\max\left\{y_0\,x_{-1},\frac{x_{-1}}{A^4}\right\}=\frac{x_{-1}}{A^4}\qquad y_{19}=\max\left\{\frac{A}{y_{15}},\frac{x_{16}}{y_{15}}\right\}=\max\left\{x_0y_{-1},\frac{y_{-1}}{A^4}\right\}=\frac{y_{-1}}{A^4}$$

$$x_{20}=\max\left\{\frac{A}{x_{16}},\frac{y_{17}}{x_{16}}\right\}=\max\left\{A^4x_0,\frac{x_0y_{-1}}{A}\right\}=A^4x_0\qquad y_{20}=\max\left\{\frac{A}{y_{16}},\frac{x_5}{y_4}\right\}=\max\left\{A^4y_0,\frac{x_{-1}y_0}{A}\right\}=A^4y_0$$

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Teorem 4.3: Eğer A<1 ise $x_{n+1}=\max\left\{\frac{A}{x_{n-1}},\frac{y_n}{x_{n-1}}\right\};y_{n+1}=\max\left\{\frac{A}{y_{n-1}},\frac{x_n}{y_{n-1}}\right\}$ denkleminin $(x_n;y_n)$ çözümleri $x_0<y_0<x_{-1}<y_{-1}<A<1$, $y_0<x_0<x_{-1}<y_{-1}<A<1$ başlangıç şartlarına göre aşağıdaki şekildedir. n=0,1,2,... için

X_n ÇÖZÜMLERİ

$$x_{10n+1}=\frac{A^{2n+1}}{x_{-1}};$$

n=0,1 için $x_{10n+2}=\frac{A}{x_0y_{-1}}$, $n\geq 2$ için $x_{10n+2}=\frac{A^{2n+2}}{x_0y_{-1}^2}$;

n=0,1,2 için $x_{10n+3}=\frac{1}{A^{2n}y_0}$, $n\geq 3$ için $x_{10n+3}=\frac{A}{x_{-1}y_0}$;

$$x_{10n+4}=\frac{y_{-1}}{A^{2n+1}};$$

n=0,1,2 için $x_{10n+5}=A^{2n+1}\cdot y_0$, $n\geq 3$ için $x_{10n+5}=\frac{x_{-1}^2y_0}{A^{2n+2}}$;

$$x_{10n+6}=\frac{A^{2n+2}}{y_{-1}};$$

n=0,1,2 için $x_{10n+7}=\frac{A}{x_{-1}y_0}$, $n\geq 3$ $x_{10n+7}=\frac{A^{2n+3}}{x_{-1}^2y_0}$;

$$\text{n=0 için } x_{10n+8} = \frac{1}{Ax_0},\ n \geq 1 \text{ için } x_{10n+8} = \frac{A}{x_0 y_{-1}};$$

$$x_{10n+9} = \frac{x_{-1}}{A^{2n+2}};$$

$$\text{n=0 için } x_{10n+10} = A^2 \cdot x_0,\ n \geq 1 \text{ için } x_{10n+10} = \frac{x_0 y_{-1}^2}{A^{2n+3}};$$

Y_n ÇÖZÜMLERİ

$$y_{10n+1} = \frac{A^{2n+1}}{y_{-1}};$$

$$\text{n=0,1,2 için } y_{10n+2} = \frac{A}{x_{-1} y_0},\ n \geq 3 \text{ için } y_{10n+2} = \frac{A^{2n+2}}{x_{-1}^2 y_0};$$

$$\text{n=0,1 için } y_{10n+3} = \frac{1}{A^{2n} x_0},\ n \geq 2 \text{ için } y_{10n+3} = \frac{A}{x_0 y_{-1}};$$

$$y_{10n+4} = \frac{x_{-1}}{A^{2n+1}};$$

$$\text{n=0 için } y_{10n+5} = Ax_0,\ n=1 \text{ için } y_{10n+5} = \frac{x_0 y_{-1}}{A},\ n \geq 2 \text{ için } y_{10n+5} = \frac{x_0 y_{-1}^2}{A^{2n+2}};$$

$$y_{10n+6} = \frac{A^{2n+2}}{x_{-1}};$$

$$\text{n=0 için } y_{10n+7} = \frac{A}{x_0 y_{-1}},\ n \geq 1 \text{ için } y_{10n+7} = \frac{A^{2n+3}}{x_0 y_{-1}^2};$$

$$\text{n=0,1,2 için } y_{10n+8} = \frac{1}{A^{2n+1} y_0},\ n \geq 3\ y_{10n+8} = \frac{A}{x_{-1} y_0};$$

$$y_{10n+9} = \frac{y_{-1}}{A^{2n+2}};$$

$$\text{n=0,1 için } y_{10n+10} = A^{2n+2} \cdot y_0,\ \text{n=2 için } y_{10n+10} = \frac{x_{-1} y_0}{A},\ n \geq 3\ y_{10n+10} = \frac{x_{-1}^2 y_0}{A^{2n+3}};$$

İspat:

Bu teoremin ispatını n nin değerleri için gösterelim.

$$x_1 = \max\left\{\frac{A}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \frac{A}{x_{-1}} \qquad y_1 = \max\left\{\frac{A}{y_{-1}}, \frac{x_0}{y_{-1}}\right\} = \frac{A}{y_{-1}}$$

$$x_2 = \max\left\{\frac{A}{x_0}, \frac{y_1}{x_0}\right\} = \max\left\{\frac{A}{x_0}, \frac{A}{y_{-1}x_0}\right\} = \frac{A}{y_{-1}x_0} \qquad y_2 = \max\left\{\frac{A}{y_0}, \frac{x_1}{y_0}\right\} = \max\left\{\frac{A}{y_0}, \frac{A}{x_{-1}y_0}\right\} = \frac{A}{x_{-1}y_0}$$

$$x_3 = \max\left\{\frac{A}{x_0}, \frac{y_1}{x_0}\right\} = \max\left\{x_{-1}, \frac{1}{y_0}\right\} = \frac{1}{y_0} \qquad y_3 = \max\left\{\frac{A}{y_1}, \frac{x_2}{y_1}\right\} = \max\left\{\frac{Ay_{-1}}{A}, \frac{Ay_{-1}}{x_0Ay_{-1}}\right\} = \frac{1}{x_0}$$

$$x_4 = \max\left\{\frac{A}{x_2}, \frac{y_3}{x_2}\right\} = \max\left\{x_0y_{-1}, \frac{y_{-1}}{A}\right\} = \frac{y_{-1}}{A} \qquad y_4 = \max\left\{\frac{A}{y_2}, \frac{x_3}{y_2}\right\} = \max\left\{y_0x_{-1}, \frac{x_{-1}}{A}\right\} = \frac{x_{-1}}{A}$$

$$x_5 = \max\left\{\frac{A}{x_3}, \frac{y_4}{x_3}\right\} = \max\left\{A\,y_0, x_{-1}\frac{y_0}{A}\right\} = A\,y_0 \qquad y_5 = \max\left\{\frac{A}{y_3}, \frac{x_4}{y_3}\right\} = \max\left\{A\,x_0, y_{-1}\frac{x_0}{A}\right\} = A\,x_0$$

$$x_6 = \max\left\{\frac{A}{x_4}, \frac{y_5}{x_4}\right\} = \max\left\{\frac{A^2}{y_{-1}}, \frac{A^2x_0}{y_{-1}}\right\} = \frac{A^2}{y_{-1}} \qquad y_6 = \max\left\{\frac{A}{y_4}, \frac{x_5}{y_4}\right\} = \max\left\{\frac{A^2}{x_{-1}}, \frac{A^2y_0}{x_{-1}}\right\} = \frac{A^2}{x_{-1}}$$

$$x_7 = \max\left\{\frac{A}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \left\{\frac{1}{y_0}, \frac{A}{x_{-1}y_0}\right\} = \frac{A}{x_{-1}y_0} \qquad y_7 = \max\left\{\frac{A}{y_5}, \frac{x_6}{y_5}\right\} = \left\{\frac{1}{x_0}, \frac{A}{x_0y_{-1}}\right\} = \frac{A}{x_0y_{-1}}$$

$$x_8 = \max\left\{\frac{A}{x_6}, \frac{y_7}{x_6}\right\} = \max\left\{\frac{y_{-1}}{A}, \frac{1}{Ax_0}\right\} = \frac{1}{Ax_0} \qquad y_8 = \max\left\{\frac{A}{y_6}, \frac{x_7}{y_6}\right\} = \max\left\{\frac{x_{-1}}{A}, \frac{1}{Ay_0}\right\} = \frac{1}{Ay_0}$$

$$x_9 = \max\left\{\frac{A}{x_7}, \frac{y_8}{x_7}\right\} = \max\left\{y_0x_{-1}, \frac{x_{-1}}{A^2}\right\} = \frac{x_{-1}}{A^2} \qquad y_9 = \max\left\{\frac{A}{y_7}, \frac{x_8}{y_7}\right\} = \left\{x_0y_{-1}, \frac{y_{-1}}{A^2}\right\} = \frac{y_{-1}}{A^2}$$

$$x_{10} = \max\left\{\frac{A}{x_8}, \frac{y_9}{x_8}\right\} = \max\left\{A^2x_0, \frac{Ax_0y_{-1}}{A^2}\right\} = A^2x_0 \qquad y_{10} = \max\left\{\frac{A}{y_8}, \frac{x_9}{y_8}\right\} = \max\left\{A^2y_0, \frac{Ay_0x_{-1}}{A^2}\right\} = A^2y_0$$

$$x_{11} = \max\left\{\frac{A}{x_9}, \frac{y_{10}}{x_9}\right\} = \max\left\{\frac{A^3}{x_{-1}}, \frac{A^4y_0}{x_{-1}}\right\} = \frac{A^3}{x_{-1}} \qquad y_{11} = \max\left\{\frac{A}{y_9}, \frac{x_{10}}{y_9}\right\} = \max\left\{\frac{A^3}{y_{-1}}, \frac{A^4x_0}{y_{-1}}\right\} = \frac{A^3}{y_{-1}}$$

$$x_{12} = \max\left\{\frac{A}{x_{10}}, \frac{y_{11}}{x_{10}}\right\} = \max\left\{\frac{1}{Ax_0}, \frac{A}{x_0y_{-1}}\right\} = \frac{A}{x_0y_{-1}} \qquad y_{12} = \max\left\{\frac{A}{y_{10}}, \frac{x_{11}}{y_{10}}\right\} = \max\left\{\frac{1}{Ay_0}, \frac{A}{y_0x_{-1}}\right\} = \frac{A}{y_0x_{-1}}$$

$$x_{13} = \max\left\{\frac{A}{x_{11}}, \frac{y_{12}}{x_{11}}\right\} = \left\{\frac{x_{-1}}{A^2}, \frac{1}{A^2y_0}\right\} = \frac{1}{A^2y_0} \qquad y_{13} = \max\left\{\frac{A}{y_{11}}, \frac{x_{12}}{y_{11}}\right\} = \left\{\frac{y_{-1}}{A^2}, \frac{1}{A^2x_0}\right\} = \frac{1}{A^2x_0}$$

$$x_{14} = \max\left\{\frac{A}{x_{12}}, \frac{y_{13}}{x_{12}}\right\} = \max\left\{x_0y_{-1}, \frac{y_{-1}}{A^3}\right\} = \frac{y_{-1}}{A^3} \qquad y_{14} = \max\left\{\frac{A}{y_{12}}, \frac{x_{13}}{y_{12}}\right\} = \max\left\{y_0x_{-1}, \frac{x_{-1}}{A^3}\right\} = \frac{x_{-1}}{A^3}$$

$$x_{15} = \max\left\{\frac{A}{x_{13}}, \frac{y_{14}}{x_{13}}\right\} = \max\left\{A^3y_0, \frac{y_0x_{-1}}{A}\right\} = A^3y_0 \qquad y_{15} = \max\left\{\frac{A}{y_{13}}, \frac{x_{14}}{y_{13}}\right\} = \max\left\{A^3x_0, \frac{x_0y_{-1}}{A}\right\} = \frac{x_0y_{-1}}{A}$$

$$x_{16} = \max\left\{\frac{A}{x_{14}}, \frac{y_{15}}{x_{14}}\right\} = \max\left\{\frac{A^4}{y_{-1}}, x_0A^2\right\} = \frac{A^4}{y_{-1}} \qquad y_{16} = \max\left\{\frac{A}{y_{14}}, \frac{x_{15}}{y_{14}}\right\} = \max\left\{\frac{A^4}{x_{-1}}, \frac{A^6y_0}{x_{-1}}\right\} = \frac{A^4}{x_{-1}}$$

$$x_{17}=\max\left\{\frac{A}{x_{15}},\frac{y_{16}}{x_{15}}\right\}=\max\left\{\frac{1}{A^2y_0},\frac{A}{y_0x_{-1}}\right\}=\frac{A}{y_0x_{-1}} \qquad y_{17}=\max\left\{\frac{A}{y_{15}},\frac{x_{16}}{y_{15}}\right\}=\max\left\{\frac{A^2}{x_0y_{-1}},\frac{A^5}{x_0y_{-1}^{\,2}}\right\}=\frac{A^5}{x_0y_{-1}^{\,2}}$$

$$x_{18}=\max\left\{\frac{A}{x_{16}},\frac{y_{17}}{x_{16}}\right\}=\max\left\{\frac{y_{-1}}{A^3},\frac{A}{x_0y_{-1}}\right\}=\frac{A}{x_0y_{-1}} \qquad y_{18}=\max\left\{\frac{A}{y_{16}},\frac{x_5}{y_4}\right\}=\max\left\{\frac{x_{-1}}{A^3},\frac{1}{y_0A^3}\right\}=\frac{1}{y_0A^3}$$

$$x_{19}=\max\left\{\frac{A}{x_{15}},\frac{y_{16}}{x_{15}}\right\}=\max\left\{y_0x_{-1},\frac{x_{-1}}{A^4}\right\}=\frac{x_{-1}}{A^4} \qquad y_{19}=\max\left\{\frac{A}{y_{17}},\frac{x_{18}}{y_{17}}\right\}=\max\left\{\frac{x_0y_{-1}^{\,2}}{A^5},\frac{y_{-1}}{A^4}\right\}=\frac{y_{-1}}{A^4}$$

$$x_{20}=\max\left\{\frac{A}{x_{16}},\frac{y_{17}}{x_{16}}\right\}=\max\left\{x_0y_{-1},\frac{x_0y_{-1}^{\,2}}{A^5}\right\}=\frac{x_0y_{-1}^{\,2}}{A^5} \qquad y_{20}=\max\left\{\frac{A}{y_{16}},\frac{x_5}{y_4}\right\}=\max\left\{A^4y_0,\frac{x_{-1}y_0}{A}\right\}=A^4y_0$$

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elde edilir.

Teorem 4.4: Eğer A<1 ise $x_{n+1}=\max\left\{\frac{A}{x_{n-1}},\frac{y_n}{x_{n-1}}\right\};\, y_{n+1}=\max\left\{\frac{A}{y_{n-1}},\frac{x_n}{y_{n-1}}\right\}$

denkleminin $(x_n;y_n)$ çözümleri $x_0<y_0<y_{-1}<x_{-1}<A<1$,

$y_0<x_0<y_{-1}<x_{-1}<A<1$ başlangıç şartlarına göre aşağıdaki şekildedir.

x_n ÇÖZÜMLERİ

$$x_{10n+1}=\frac{A^{2n+1}}{x_{-1}};$$

n=0,1,2 için $x_{10n+2}=\frac{A}{x_0y_{-1}}$, $n\geq 3$ için $x_{10n+2}=\frac{A^{2n+2}}{x_0y_{-1}^2}$;

n=0,1 için $x_{10n+3}=\frac{1}{A^{2n}y_0}$, $n\geq 2$ için $x_{10n+3}=\frac{A}{x_{-1}y_0}$;

$$x_{10n+4}=\frac{y_{-1}}{A^{2n+1}};$$

n=0 için $x_{10n+5}=A\cdot y_0$, n=1 için $x_{10n+5}=\frac{x_{-1}y_0}{A}$, $n\geq 2$ için $x_{10n+5}=\frac{x_{-1}^2y_0}{A^{2n+2}}$;

$$x_{10n+6}=\frac{A^{2n+2}}{y_{-1}};$$

n=0 için $x_{10n+7}=\frac{A}{x_{-1}y_0}$, $n\geq 1$ için $x_{10n+7}=\frac{A^{2n+3}}{x_{-1}^2 y_0}$;

n=0,1,2 için $x_{10n+8}=\frac{1}{A^{2n+1}x_0}$, $n\geq 3$ için $x_{10n+8}=\frac{A}{x_0 y_{-1}}$;

$$x_{10n+9}=\frac{x_{-1}}{A^{2n+2}};$$

n=0,1 için $x_{10n+10}=A^{2n+2}\cdot x_0$, $n=2$ için $x_{10n+10}=\frac{x_0 y_{-1}}{A}$, $n\geq 3$ için

$$x_{10n+10}=\frac{x_0 y_{-1}^2}{A^{2n+3}};$$

Y_n ÇÖZÜMLERİ

$$y_{10n+1}=\frac{A^{2n+1}}{y_{-1}};$$

n=0,1 için $y_{10n+2}=\frac{A}{x_{-1}y_0}$, $n\geq 2$ için $y_{10n+2}=\frac{A^{2n+2}}{x_{-1}^2 y_0}$;

n=0,1,2 için $y_{10n+3}=\frac{1}{A^{2n}x_0}$, $n\geq 3$ için $y_{10n+3}=\frac{A}{x_0 y_{-1}}$;

$$y_{10n+4}=\frac{x_{-1}}{A^{2n+1}};$$

n=0,1,2 için $y_{10n+5}=A^{2n+1}\cdot x_0$, $n\geq 3$ iç in $y_{10n+5}=\frac{x_0 y_{-1}^2}{A^{2n+2}}$;

$$y_{10n+6}=\frac{A^{2n+2}}{x_{-1}};$$

n=0,1,2 için $y_{10n+7}=\frac{A}{x_0 y_{-1}}$, $n\geq 3$ için $y_{10n+7}=\frac{A^{2n+3}}{x_0 y_{-1}^2}$;

n=0 için $y_{10n+8}=\frac{1}{Ay_0}$, $n\geq 1$ için $y_{10n+8}=\frac{A}{x_{-1}y_0}$;

$$y_{10n+9}=\frac{y_{-1}}{A^{2n+2}};$$

n=0 için $y_{10n+10}=A^2\cdot y_0$, $n\geq 1$ için $y_{10n+10}=\frac{x_{-1}^2 y_0}{A^{2n+3}}$;

İspat:

Bu teoremin ispatını n nin değerleri için gösterelim.

$$x_1 = \max\left\{\frac{A}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \frac{A}{x_{-1}} \qquad y_1 = \max\left\{\frac{A}{y_{-1}}, \frac{x_0}{y_{-1}}\right\} = \frac{A}{y_{-1}}$$

$$x_2 = \max\left\{\frac{A}{x_0}, \frac{y_1}{x_0}\right\} = \max\left\{\frac{A}{x_0}, \frac{A}{y_{-1}x_0}\right\} = \frac{A}{y_{-1}x_0} \qquad y_2 = \max\left\{\frac{A}{y_0}, \frac{x_1}{y_0}\right\} = \max\left\{\frac{A}{y_0}, \frac{A}{x_{-1}y_0}\right\} = \frac{A}{x_{-1}y_0}$$

$$x_3 = \max\left\{\frac{A}{x_0}, \frac{y_1}{x_0}\right\} = \max\left\{x_{-1}, \frac{1}{y_0}\right\} = \frac{1}{y_0} \qquad y_3 = \max\left\{\frac{A}{y_1}, \frac{x_2}{y_1}\right\} = \max\left\{\frac{Ay_{-1}}{A}, \frac{Ay_{-1}}{x_0 A y_{-1}}\right\} = \frac{1}{x_0}$$

$$x_4 = \max\left\{\frac{A}{x_2}, \frac{y_3}{x_2}\right\} = \max\left\{x_0 y_{-1}, \frac{y_{-1}}{A}\right\} = \frac{y_{-1}}{A} \qquad y_4 = \max\left\{\frac{A}{y_2}, \frac{x_3}{y_2}\right\} = \max\left\{y_0 x_{-1}, \frac{x_{-1}}{A}\right\} = \frac{x_{-1}}{A}$$

$$x_5 = \max\left\{\frac{A}{x_3}, \frac{y_4}{x_3}\right\} = \max\left\{A\, y_0, x_{-1}\frac{y_0}{A}\right\} = A\, y_0 \qquad y_5 = \max\left\{\frac{A}{y_3}, \frac{x_4}{y_3}\right\} = \max\left\{A\, x_0, y_{-1}\frac{x_0}{A}\right\} = A\, x_0$$

$$x_6 = \max\left\{\frac{A}{x_4}, \frac{y_5}{x_4}\right\} = \max\left\{\frac{A^2}{y_{-1}}, \frac{A^2 x_0}{y_{-1}}\right\} = \frac{A^2}{y_{-1}} \qquad y_6 = \max\left\{\frac{A}{y_4}, \frac{x_5}{y_4}\right\} = \max\left\{\frac{A^2}{x_{-1}}, \frac{A^2 y_0}{x_{-1}}\right\} = \frac{A^2}{x_{-1}}$$

$$x_7 = \max\left\{\frac{A}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \left\{\frac{1}{y_0}, \frac{A}{x_{-1}y_0}\right\} = \frac{A}{x_{-1}y_0} \qquad y_7 = \max\left\{\frac{A}{y_5}, \frac{x_6}{y_5}\right\} = \left\{\frac{1}{x_0}, \frac{A}{x_0 y_{-1}}\right\} = \frac{A}{x_0 y_{-1}}$$

$$x_8 = \max\left\{\frac{A}{x_6}, \frac{y_7}{x_6}\right\} = \max\left\{\frac{y_{-1}}{A}, \frac{1}{Ax_0}\right\} = \frac{1}{Ax_0} \qquad y_8 = \max\left\{\frac{A}{y_6}, \frac{x_7}{y_6}\right\} = \max\left\{\frac{x_{-1}}{A}, \frac{1}{Ay_0}\right\} = \frac{1}{Ay_0}$$

$$x_9 = \max\left\{\frac{A}{x_7}, \frac{y_8}{x_7}\right\} = \max\left\{y_0 x_{-1}, \frac{x_{-1}}{A^2}\right\} = \frac{x_{-1}}{A^2} \qquad y_9 = \max\left\{\frac{A}{y_7}, \frac{x_8}{y_7}\right\} = \left\{x_0 y_{-1}, \frac{y_{-1}}{A^2}\right\} = \frac{y_{-1}}{A^2}$$

$$x_{10} = \max\left\{\frac{A}{x_8}, \frac{y_9}{x_8}\right\} = \max\left\{A^2 x_0, \frac{Ax_0 y_{-1}}{A^2}\right\} = A^2 x_0 \qquad y_{10} = \max\left\{\frac{A}{y_8}, \frac{x_9}{y_8}\right\} = \max\left\{A^2 y_0, \frac{Ay_0 x_{-1}}{A^2}\right\} = A^2 y_0$$

$$x_{11} = \max\left\{\frac{A}{x_9}, \frac{y_{10}}{x_9}\right\} = \max\left\{\frac{A^3}{x_{-1}}, \frac{A^4 y_0}{x_{-1}}\right\} = \frac{A^3}{x_{-1}} \qquad y_{11} = \max\left\{\frac{A}{y_9}, \frac{x_{10}}{y_9}\right\} = \max\left\{\frac{A^3}{y_{-1}}, \frac{A^4 x_0}{y_{-1}}\right\} = \frac{A^3}{y_{-1}}$$

$$x_{12} = \max\left\{\frac{A}{x_{10}}, \frac{y_{11}}{x_{10}}\right\} = \max\left\{\frac{1}{Ax_0}, \frac{A}{x_0 y_{-1}}\right\} = \frac{A}{x_0 y_{-1}} \qquad y_{12} = \max\left\{\frac{A}{y_{10}}, \frac{x_{11}}{y_{10}}\right\} = \max\left\{\frac{1}{A^2 y_0}, \frac{A}{y_0 x_{-1}}\right\} = \frac{A}{y_0 x_{-1}}$$

$$x_{13} = \max\left\{\frac{A}{x_{11}}, \frac{y_{12}}{x_{11}}\right\} = \left\{\frac{x_{-1}}{A^2}, \frac{1}{A^2 y_0}\right\} = \frac{1}{A^2 y_0} \qquad y_{13} = \max\left\{\frac{A}{y_{11}}, \frac{x_{12}}{y_{11}}\right\} = \left\{\frac{y_{-1}}{A^2}, \frac{1}{A^2 x_0}\right\} = \frac{1}{A^2 x_0}$$

$$x_{14} = \max\left\{\frac{A}{x_{12}}, \frac{y_{13}}{x_{12}}\right\} = \max\left\{x_0 y_{-1}, \frac{y_{-1}}{A^3}\right\} = \frac{y_{-1}}{A^3} \qquad y_{14} = \max\left\{\frac{A}{y_{12}}, \frac{x_{13}}{y_{12}}\right\} = \max\left\{y_0 x_{-1}, \frac{x_{-1}}{A^3}\right\} = \frac{x_{-1}}{A^3}$$

$$x_{15}=\max\left\{\frac{A}{x_{13}},\frac{y_{14}}{x_{13}}\right\}=\max\left\{A^3y_0,\frac{y_0x_{-1}}{A}\right\}=\frac{y_0x_{-1}}{A}\qquad y_{15}=\max\left\{\frac{A}{y_{13}},\frac{x_{14}}{y_{13}}\right\}=\max\left\{A^3x_0,\frac{x_0y_{-1}}{A}\right\}=A^3x_0$$

$$x_{16}=\max\left\{\frac{A}{x_{14}},\frac{y_{15}}{x_{14}}\right\}=\max\left\{\frac{A^4}{y_{-1}},\frac{A^6x_0}{y_{-1}}\right\}=\frac{A^4}{y_{-1}}\qquad y_{16}=\max\left\{\frac{A}{y_{14}},\frac{x_{15}}{y_{14}}\right\}=\max\left\{\frac{A^4}{x_{-1}},\frac{A^6y_0}{x_{-1}}\right\}=\frac{A^4}{x_{-1}}$$

$$x_{17}=\max\left\{\frac{A}{x_{15}},\frac{y_{16}}{x_{15}}\right\}=\max\left\{\frac{A^2}{y_0x_{-1}},\frac{A^5}{y_0x_{-1}^2}\right\}=\frac{A^5}{y_0x_{-1}^2}\qquad y_{17}=\max\left\{\frac{A}{y_{15}},\frac{x_{16}}{y_{15}}\right\}=\max\left\{\frac{A^2}{x_0y_{-1}},\frac{A^5}{x_0y_{-1}^2}\right\}=\frac{A}{x_0y_{-1}}$$

$$x_{18}=\max\left\{\frac{A}{x_{16}},\frac{y_{17}}{x_{16}}\right\}=\max\left\{\frac{y_{-1}}{A^3},\frac{1}{x_0A^3}\right\}=\frac{1}{x_0A^3}\qquad y_{18}=\max\left\{\frac{A}{y_{16}},\frac{x_5}{y_4}\right\}=\max\left\{\frac{x_{-1}}{A^3},\frac{A}{y_0x_{-1}}\right\}=\frac{A}{y_0x_{-1}}$$

$$x_{19}=\max\left\{\frac{A}{x_{15}},\frac{y_{16}}{x_{15}}\right\}=\max\left\{\frac{y_0x_{-1}^2}{A^4},\frac{x_{-1}}{A^4}\right\}=\frac{x_{-1}}{A^4}\qquad y_{19}=\max\left\{\frac{A}{y_{15}},\frac{x_{16}}{y_{15}}\right\}=\max\left\{x_0y_{-1},\frac{y_{-1}}{A^4}\right\}=\frac{y_{-1}}{A^4}$$

$$x_{20}=\max\left\{\frac{A}{x_{16}},\frac{y_{17}}{x_{16}}\right\}=\max\left\{A^4x_0,\frac{x_0y_{-1}}{A}\right\}=A^4x_0\qquad y_{20}=\max\left\{\frac{A}{y_{16}},\frac{x_5}{y_4}\right\}=\max\left\{y_0x_{-1},\frac{y_0x_{-1}^2}{A^5}\right\}=\frac{y_0x_{-1}^2}{A^5}$$

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Teorem 4.5: Eğer A<1 ise $x_{n+1}=\max\left\{\frac{A}{x_{n-1}},\frac{y_n}{x_{n-1}}\right\};y_{n+1}=\max\left\{\frac{A}{y_{n-1}},\frac{x_n}{y_{n-1}}\right\}$ denkleminin $(x_n;y_n)$ çözümleri $x_0<y_{-1}<x_{-1}<y_0<A<1, y_0<y_{-1}<x_{-1}<x_0<A<1$ başlangıç şartlarına göre aşağıdaki şekildedir:

X_n ÇÖZÜMLERİ

$$x_{10n+1}=\frac{A^{2n+1}}{x_{-1}};$$

$$x_{10n+2}=\frac{A}{x_0y_{-1}};$$

n=0,1,2 için $x_{10n+3}=\frac{1}{A^{2n}y_0}$, $n\geq 3$ için $x_{10n+3}=\frac{A}{x_{-1}y_0}$;

$$x_{10n+4}=\frac{y_{-1}}{A^{2n+1}};$$

n=0,1,2 için $x_{10n+5}=A^{2n+1}\cdot y_0$, $n\geq 3$ için $x_{10n+5}=\frac{x_{-1}^2 y_0}{A^{2n+2}}$;

$$x_{10n+6}=\frac{A^{2n+2}}{y_{-1}};$$

n=0,1,2 için $x_{10n+7}=\frac{A}{x_{-1}y_0}$, $n\geq 3$ için $x_{10n+7}=\frac{A^{2n+3}}{x_{-1}^2 y_0}$;

n=0,1,2,3,4 için $x_{10n+8}=\frac{1}{A^{2n+1}x_0}$, $n\geq 5$ için $x_{10n+8}=\frac{A}{x_0 y_{-1}}$;

$$x_{10n+9}=\frac{x_{-1}}{A^{2n+2}};$$

n=0,1,2,3,4 için $x_{10n+10}=A^{2n+2}\cdot x_0$, $n\geq 5$ için $x_{10n+10}=\frac{x_0 y_{-1}^2}{A^{2n+3}}$;

Y_n ÇÖZÜMLERİ

$$y_{10n+1}=\frac{A^{2n+1}}{y_{-1}};$$

n=0,1,2 için $y_{10n+2}=\frac{A}{x_{-1}y_0}$, $n\geq 3$ için $y_{10n+2}=\frac{A^{2n+2}}{x_{-1}^2 y_0}$;

$$y_{10n+3}=\frac{1}{A^{2n}x_0};$$

$$y_{10n+4}=\frac{x_{-1}}{A^{2n+1}};$$

n=0,1,2,3,4 için $y_{10n+5}=A^{2n+1}\cdot x_0$, $n\geq 5$ için $y_{10n+5}=\frac{x_0 y_{-1}}{A}$;

$$y_{10n+6}=\frac{A^{2n+2}}{x_{-1}};$$

n=0,1,2,3,4 için $y_{10n+7}=\frac{A}{x_0 y_{-1}}$, $n\geq 5$ için $y_{10n+7}=\frac{A^{2n+3}}{x_0 y_{-1}^2}$;

n=0,1,2 için $y_{10n+8}=\frac{1}{A^{2n+1}y_0}$, $n\geq 3$ için $y_{10n+8}=\frac{A}{x_{-1}y_0}$;

$$y_{10n+9} = \frac{y_{-1}}{A^{2n+2}};$$

n=0,1 için $y_{10n+10} = A^{2n+2} \cdot y_0$, n=2 için $y_{10n+10} = \frac{x_{-1}y_0}{A}$, $n \geq 3$ için

$$y_{10n+10} = \frac{x_{-1}^2 y_0}{A^{2n+3}};$$

İspat:

Bu teoremin ispatını n nin değerleri için gösterelim.

$$x_1 = \max\left\{\frac{A}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \frac{A}{x_{-1}} \qquad y_1 = \max\left\{\frac{A}{y_{-1}}, \frac{x_0}{y_{-1}}\right\} = \frac{A}{y_{-1}}$$

$$x_2 = \max\left\{\frac{A}{x_0}, \frac{y_1}{x_0}\right\} = \max\left\{\frac{A}{x_0}, \frac{A}{y_{-1}x_0}\right\} = \frac{A}{y_{-1}x_0} \qquad y_2 = \max\left\{\frac{A}{y_0}, \frac{x_1}{y_0}\right\} = \max\left\{\frac{A}{y_0}, \frac{A}{x_{-1}y_0}\right\} = \frac{A}{x_{-1}y_0}$$

$$x_3 = \max\left\{\frac{A}{x_0}, \frac{y_1}{x_0}\right\} = \max\left\{x_{-1}, \frac{1}{y_0}\right\} = \frac{1}{y_0} \qquad y_3 = \max\left\{\frac{A}{y_1}, \frac{x_2}{y_1}\right\} = \max\left\{\frac{Ay_{-1}}{A}, \frac{Ay_{-1}}{x_0 A y_{-1}}\right\} = \frac{1}{x_0}$$

$$x_4 = \max\left\{\frac{A}{x_2}, \frac{y_3}{x_2}\right\} = \max\left\{x_0 y_{-1}, \frac{y_{-1}}{A}\right\} = \frac{y_{-1}}{A} \qquad y_4 = \max\left\{\frac{A}{y_2}, \frac{x_3}{y_2}\right\} = \max\left\{y_0 x_{-1}, \frac{x_{-1}}{A}\right\} = \frac{x_{-1}}{A}$$

$$x_5 = \max\left\{\frac{A}{x_3}, \frac{y_4}{x_3}\right\} = \max\left\{A\, y_0, x_{-1}\frac{y_0}{A}\right\} = A\, y_0 \qquad y_5 = \max\left\{\frac{A}{y_3}, \frac{x_4}{y_3}\right\} = \max\left\{A\, x_0, y_{-1}\frac{x_0}{A}\right\} = A\, x_0$$

$$x_6 = \max\left\{\frac{A}{x_4}, \frac{y_5}{x_4}\right\} = \max\left\{\frac{A^2}{y_{-1}}, \frac{A^2 x_0}{y_{-1}}\right\} = \frac{A^2}{y_{-1}} \qquad y_6 = \max\left\{\frac{A}{y_4}, \frac{x_5}{y_4}\right\} = \max\left\{\frac{A^2}{x_{-1}}, \frac{A^2 y_0}{x_{-1}}\right\} = \frac{A^2}{x_{-1}}$$

$$x_7 = \max\left\{\frac{A}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \left\{\frac{1}{y_0}, \frac{A}{x_{-1}y_0}\right\} = \frac{A}{x_{-1}y_0} \qquad y_7 = \max\left\{\frac{A}{y_5}, \frac{x_6}{y_5}\right\} = \left\{\frac{1}{x_0}, \frac{A}{x_0 y_{-1}}\right\} = \frac{A}{x_0 y_{-1}}$$

$$x_8 = \max\left\{\frac{A}{x_6}, \frac{y_7}{x_6}\right\} = \max\left\{\frac{y_{-1}}{A}, \frac{1}{Ax_0}\right\} = \frac{1}{Ax_0} \qquad y_8 = \max\left\{\frac{A}{y_6}, \frac{x_7}{y_6}\right\} = \max\left\{\frac{x_{-1}}{A}, \frac{1}{Ay_0}\right\} = \frac{1}{Ay_0}$$

$$x_9 = \max\left\{\frac{A}{x_7}, \frac{y_8}{x_7}\right\} = \max\left\{y_0 x_{-1}, \frac{x_{-1}}{A^2}\right\} = \frac{x_{-1}}{A^2} \qquad y_9 = \max\left\{\frac{A}{y_7}, \frac{x_8}{y_7}\right\} = \left\{x_0 y_{-1}, \frac{y_{-1}}{A^2}\right\} = \frac{y_{-1}}{A^2}$$

$$x_{10} = \max\left\{\frac{A}{x_8}, \frac{y_9}{x_8}\right\} = \max\left\{A^2 x_0, \frac{Ax_0 y_{-1}}{A^2}\right\} = A^2 x_0 \qquad y_{10} = \max\left\{\frac{A}{y_8}, \frac{x_9}{y_8}\right\} = \max\left\{A^2 y_0, \frac{Ay_0 x_{-1}}{A^2}\right\} = A^2 y_0$$

$$x_{11} = \max\left\{\frac{A}{x_9}, \frac{y_{10}}{x_9}\right\} = \max\left\{\frac{A^3}{x_{-1}}, \frac{A^4 y_0}{x_{-1}}\right\} = \frac{A^3}{x_{-1}} \qquad y_{11} = \max\left\{\frac{A}{y_9}, \frac{x_{10}}{y_9}\right\} = \max\left\{\frac{A^3}{y_{-1}}, \frac{A^4 x_0}{y_{-1}}\right\} = \frac{A^3}{y_{-1}}$$

$$x_{12}=\max\left\{\frac{A}{x_{10}},\frac{y_{11}}{x_{10}}\right\}=\max\left\{\frac{1}{Ax_0},\frac{A}{x_0y_{-1}}\right\}=\frac{A}{x_0y_{-1}} \qquad y_{12}=\max\left\{\frac{A}{y_{10}},\frac{x_{11}}{y_{10}}\right\}=\max\left\{\frac{1}{Ay_0},\frac{A}{y_0x_{-1}}\right\}=\frac{A}{y_0x_{-1}}$$

$$x_{13}=\max\left\{\frac{A}{x_{11}},\frac{y_{12}}{x_{11}}\right\}=\left\{\frac{x_{-1}}{A^2},\frac{1}{A^2y_0}\right\}=\frac{1}{A^2y_0} \qquad y_{13}=\max\left\{\frac{A}{y_{11}},\frac{x_{12}}{y_{11}}\right\}=\left\{\frac{y_{-1}}{A^2},\frac{1}{A^2x_0}\right\}=\frac{1}{A^2x_0}$$

$$x_{14}=\max\left\{\frac{A}{x_{12}},\frac{y_{13}}{x_{12}}\right\}=\max\left\{x_0y_{-1},\frac{y_{-1}}{A^3}\right\}=\frac{y_{-1}}{A^3} \qquad y_{14}=\max\left\{\frac{A}{y_{12}},\frac{x_{13}}{y_{12}}\right\}=\max\left\{y_0x_{-1},\frac{x_{-1}}{A^3}\right\}=\frac{x_{-1}}{A^3}$$

$$x_{15}=\max\left\{\frac{A}{x_{13}},\frac{y_{14}}{x_{13}}\right\}=\max\left\{A^3y_0,\frac{y_0x_{-1}}{A}\right\}=A^3y_0 \qquad y_{15}=\max\left\{\frac{A}{y_{13}},\frac{x_{14}}{y_{13}}\right\}=\max\left\{A^3x_0,\frac{x_0y_{-1}}{A}\right\}=A^3x_0$$

$$x_{16}=\max\left\{\frac{A}{x_{14}},\frac{y_{15}}{x_{14}}\right\}=\max\left\{\frac{A^4}{y_{-1}},\frac{x_0A^6}{y_{-1}}\right\}=\frac{A^4}{y_{-1}} \qquad y_{16}=\max\left\{\frac{A}{y_{14}},\frac{x_{15}}{y_{14}}\right\}=\max\left\{\frac{A^4}{x_{-1}},\frac{A^6y_0}{x_{-1}}\right\}=\frac{A^4}{x_{-1}}$$

$$x_{17}=\max\left\{\frac{A}{x_{15}},\frac{y_{16}}{x_{15}}\right\}=\max\left\{\frac{1}{A^2y_0},\frac{A}{y_0x_{-1}}\right\}=\frac{A}{y_0x_{-1}} \qquad y_{17}=\max\left\{\frac{A}{y_{15}},\frac{x_{16}}{y_{15}}\right\}=\max\left\{\frac{1}{x_0A^2},\frac{A}{x_0y_{-1}}\right\}=\frac{A}{x_0y_{-1}}$$

$$x_{18}=\max\left\{\frac{A}{x_{16}},\frac{y_{17}}{x_{16}}\right\}=\max\left\{\frac{y_{-1}}{A^3},\frac{1}{x_0A^3}\right\}=\frac{1}{x_0A^3} \qquad y_{18}=\max\left\{\frac{A}{y_{16}},\frac{x_5}{y_4}\right\}=\max\left\{\frac{x_{-1}}{A^3},\frac{1}{y_0A^3}\right\}=\frac{1}{y_0A^3}$$

$$x_{19}=\max\left\{\frac{A}{x_{15}},\frac{y_{16}}{x_{15}}\right\}=\max\left\{y_0x_{-1},\frac{x_{-1}}{A^4}\right\}=\frac{x_{-1}}{A^4} \qquad y_{19}=\max\left\{\frac{A}{y_{15}},\frac{x_{16}}{y_{15}}\right\}=\max\left\{x_0y_{-1},\frac{y_{-1}}{A^4}\right\}=\frac{y_{-1}}{A^4}$$

$$x_{20}=\max\left\{\frac{A}{x_{16}},\frac{y_{17}}{x_{16}}\right\}=\max\left\{A^4x_0,\frac{x_0y_{-1}}{A}\right\}=A^4x_0 \qquad y_{20}=\max\left\{\frac{A}{y_{16}},\frac{x_5}{y_4}\right\}=\max\left\{A^4y_0,\frac{x_{-1}y_0}{A}\right\}=A^4y_0$$

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Teorem 4.6:Eğer A<1 ise $x_{n+1}=\max\left\{\frac{A}{x_{n-1}},\frac{y_n}{x_{n-1}}\right\}; y_{n+1}=\max\left\{\frac{A}{y_{n-1}},\frac{x_n}{y_{n-1}}\right\}$

denkleminin $(x_n; y_n)$ çözümleri $x_0<y_{-1}<y_0<x_{-1}<A<1$,

$y_0<y_{-1}<x_0<x_{-1}<A<1$ başlangıç şartlarına göre aşağıdaki şekildedir.

X_n ÇÖZÜMLERİ

$$x_{10n+1}=\frac{A^{2n+1}}{x_{-1}};$$

$$x_{10n+2}=\frac{A}{x_0 y_{-1}};$$

n=0,1 için $x_{10n+3}=\frac{1}{A^{2n}y_0}$, $n\geq 2$ için $x_{10n+3}=\frac{A}{x_{-1}y_0}$;

$$x_{10n+4}=\frac{y_{-1}}{A^{2n+1}};$$

n=0 için $x_{10n+5}=A\cdot y_0$, n=1 için $x_{10n+5}=\frac{x_{-1}y_0}{A}$, $n\geq 2$ için

$$x_{10n+5}=\frac{x_{-1}^2 y_0}{A^{2n+2}};$$

$$x_{10n+6}=\frac{A^{2n+2}}{y_{-1}};$$

n=0 için $x_{10n+7}=\frac{A}{x_{-1}y_0}$, $n\geq 1$ için $x_{10n+7}=\frac{A^{2n+3}}{x_{-1}^2 y_0}$;

$n=0,1,2,3,4$ $x_{10n+8}=\frac{1}{A^{2n+1}x_0}$, $n\geq 5$ $x_{10n+8}=\frac{A}{x_0 y_{-1}}$;

$$x_{10n+9}=\frac{x_{-1}}{A^{2n+2}};$$

$n=0,1,2,3,4$ $x_{10n+10}=A^{2n+2}\cdot x_0$, $n\geq 5$ $x_{10n+10}=\frac{x_0 y_{-1}^2}{A^{2n+3}}$,

Y_n ÇÖZÜMLERİ

$$y_{10n+1}=\frac{A^{2n+1}}{y_{-1}};$$

n=0,1 için $y_{10n+2}=\frac{A}{x_{-1}y_0}$, $n\geq 2$ için $y_{10n+2}=\frac{A^{2n+2}}{x_{-1}^2 y_0}$;

$$y_{10n+3}=\frac{1}{A^{2n}x_0};$$

$$y_{10n+4}=\frac{x_{-1}}{A^{2n+1}};$$

n=0,1,2,3,4 için $y_{10n+5}=A^{2n+1}\cdot x_0$, $n\geq 5$ $y_{10n+5}=\frac{x_0 y_{-1}}{A}$;

$$y_{10n+6}=\frac{A^{2n+2}}{x_{-1}};$$

n=0,1,2,3,4 için $y_{10n+7}=\frac{A}{x_0y_{-1}}$, $n\geq 5$ $y_{10n+7}=\frac{A^{2n+3}}{x_0y_{-1}^2}$;

n=0 için $y_{10n+8}=\frac{1}{Ay_0}$, $n\geq 1$ için $y_{10n+8}=\frac{A}{x_{-1}y_0}$;

$$y_{10n+9}=\frac{y_{-1}}{A^{2n+2}};$$

n=0 için $y_{10n+10}=A^2\cdot y_0$, $n\geq 1$ için $y_{10n+10}=\frac{x_{-1}^2y_0}{A^{2n+3}}$;

İspat:

Bu teoremin ispatını n nin değerleri için gösterelim.

$$x_1=\max\left\{\frac{A}{x_{-1}},\frac{y_0}{x_{-1}}\right\}=\frac{A}{x_{-1}} \qquad y_1=\max\left\{\frac{A}{y_{-1}},\frac{x_0}{y_{-1}}\right\}=\frac{A}{y_{-1}}$$

$$x_2=\max\left\{\frac{A}{x_0},\frac{y_1}{x_0}\right\}=\max\left\{\frac{A}{x_0},\frac{A}{y_{-1}x_0}\right\}=\frac{A}{y_{-1}x_0} \qquad y_2=\max\left\{\frac{A}{y_0},\frac{x_1}{y_0}\right\}=\max\left\{\frac{A}{y_0},\frac{A}{x_{-1}y_0}\right\}=\frac{A}{x_{-1}y_0}$$

$$x_3=\max\left\{\frac{A}{x_0},\frac{y_1}{x_0}\right\}=\max\left\{x_{-1},\frac{1}{y_0}\right\}=\frac{1}{y_0} \qquad y_3=\max\left\{\frac{A}{y_1},\frac{x_2}{y_1}\right\}=\max\left\{\frac{Ay_{-1}}{A},\frac{Ay_{-1}}{x_0Ay_{-1}}\right\}=\frac{1}{x_0}$$

$$x_4=\max\left\{\frac{A}{x_2},\frac{y_3}{x_2}\right\}=\max\left\{x_0y_{-1},\frac{y_{-1}}{A}\right\}=\frac{y_{-1}}{A} \qquad y_4=\max\left\{\frac{A}{y_2},\frac{x_3}{y_2}\right\}=\max\left\{y_0x_{-1},\frac{x_{-1}}{A}\right\}=\frac{x_{-1}}{A}$$

$$x_5=\max\left\{\frac{A}{x_3},\frac{y_4}{x_3}\right\}=\max\left\{A\,y_0,x_{-1}\frac{y_0}{A}\right\}=A\,y_0 \qquad y_5=\max\left\{\frac{A}{y_3},\frac{x_4}{y_3}\right\}=\max\left\{A\,x_0,y_{-1}\frac{x_0}{A}\right\}=A\,x_0$$

$$x_6=\max\left\{\frac{A}{x_4},\frac{y_5}{x_4}\right\}=\max\left\{\frac{A^2}{y_{-1}},\frac{A^2x_0}{y_{-1}}\right\}=\frac{A^2}{y_{-1}} \qquad y_6=\max\left\{\frac{A}{y_4},\frac{x_5}{y_4}\right\}=\max\left\{\frac{A^2}{x_{-1}},\frac{A^2y_0}{x_{-1}}\right\}=\frac{A^2}{x_{-1}}$$

$$x_7=\max\left\{\frac{A}{x_{-1}},\frac{y_0}{x_{-1}}\right\}=\left\{\frac{1}{y_0},\frac{A}{x_{-1}y_0}\right\}=\frac{A}{x_{-1}y_0} \qquad y_7=\max\left\{\frac{A}{y_5},\frac{x_6}{y_5}\right\}=\left\{\frac{1}{x_0},\frac{A}{x_0y_{-1}}\right\}=\frac{A}{x_0y_{-1}}$$

$$x_8=\max\left\{\frac{A}{x_6},\frac{y_7}{x_6}\right\}=\max\left\{\frac{y_{-1}}{A},\frac{1}{Ax_0}\right\}=\frac{1}{Ax_0} \qquad y_8=\max\left\{\frac{A}{y_6},\frac{x_7}{y_6}\right\}=\max\left\{\frac{x_{-1}}{A},\frac{1}{Ay_0}\right\}=\frac{1}{Ay_0}$$

$$x_9 = \max\left\{\frac{A}{x_7},\frac{y_8}{x_7}\right\} = \max\left\{y_0 x_{-1},\frac{x_{-1}}{A^2}\right\} = \frac{x_{-1}}{A^2} \qquad y_9 = \max\left\{\frac{A}{y_7},\frac{x_8}{y_7}\right\} = \left\{x_0 y_{-1},\frac{y_{-1}}{A^2}\right\} = \frac{y_{-1}}{A^2}$$

$$x_{10} = \max\left\{\frac{A}{x_8},\frac{y_9}{x_8}\right\} = \max\left\{A^2 x_0,\frac{A x_0 y_{-1}}{A^2}\right\} = A^2 x_0 \qquad y_{10} = \max\left\{\frac{A}{y_8},\frac{x_9}{y_8}\right\} = \max\left\{A^2 y_0,\frac{A y_0 x_{-1}}{A^2}\right\} = A^2 y_0$$

$$x_{11} = \max\left\{\frac{A}{x_9},\frac{y_{10}}{x_9}\right\} = \max\left\{\frac{A^3}{x_{-1}},\frac{A^4 y_0}{x_{-1}}\right\} = \frac{A^3}{x_{-1}} \qquad y_{11} = \max\left\{\frac{A}{y_9},\frac{x_{10}}{y_9}\right\} = \max\left\{\frac{A^3}{y_{-1}},\frac{A^4 x_0}{y_{-1}}\right\} = \frac{A^3}{y_{-1}}$$

$$x_{12} = \max\left\{\frac{A}{x_{10}},\frac{y_{11}}{x_{10}}\right\} = \max\left\{\frac{1}{A x_0},\frac{A}{x_0 y_{-1}}\right\} = \frac{A}{x_0 y_{-1}} \qquad y_{12} = \max\left\{\frac{A}{y_{10}},\frac{x_{11}}{y_{10}}\right\} = \max\left\{\frac{1}{A^2 y_0},\frac{A}{y_0 x_{-1}}\right\} = \frac{A}{y_0 x_{-1}}$$

$$x_{13} = \max\left\{\frac{A}{x_{11}},\frac{y_{12}}{x_{11}}\right\} = \left\{\frac{x_{-1}}{A^2},\frac{1}{A^2 y_0}\right\} = \frac{1}{A^2 y_0} \qquad y_{13} = \max\left\{\frac{A}{y_{11}},\frac{x_{12}}{y_{11}}\right\} = \left\{\frac{y_{-1}}{A^2},\frac{1}{A^2 x_0}\right\} = \frac{1}{A^2 x_0}$$

$$x_{14} = \max\left\{\frac{A}{x_{12}},\frac{y_{13}}{x_{12}}\right\} = \max\left\{x_0 y_{-1},\frac{y_{-1}}{A^3}\right\} = \frac{y_{-1}}{A^3} \qquad y_{14} = \max\left\{\frac{A}{y_{12}},\frac{x_{13}}{y_{12}}\right\} = \max\left\{y_0 x_{-1},\frac{x_{-1}}{A^3}\right\} = \frac{x_{-1}}{A^3}$$

$$x_{15} = \max\left\{\frac{A}{x_{13}},\frac{y_{14}}{x_{13}}\right\} = \max\left\{A^3 y_0,\frac{y_0 x_{-1}}{A}\right\} = \frac{y_0 x_{-1}}{A} \qquad y_{15} = \max\left\{\frac{A}{y_{13}},\frac{x_{14}}{y_{13}}\right\} = \max\left\{A^3 x_0,\frac{x_0 y_{-1}}{A}\right\} = A^3 x_0$$

$$x_{16} = \max\left\{\frac{A}{x_{14}},\frac{y_{15}}{x_{14}}\right\} = \max\left\{\frac{A^4}{y_{-1}},\frac{A^6 x_0}{y_{-1}}\right\} = \frac{A^4}{y_{-1}} \qquad y_{16} = \max\left\{\frac{A}{y_{14}},\frac{x_{15}}{y_{14}}\right\} = \max\left\{\frac{A^4}{x_{-1}},\frac{A^6 y_0}{x_{-1}}\right\} = \frac{A^4}{x_{-1}}$$

$$x_{17} = \max\left\{\frac{A}{x_{15}},\frac{y_{16}}{x_{15}}\right\} = \max\left\{\frac{A^2}{y_0 x_{-1}},\frac{A^5}{y_0 x_{-1}^{\ 2}}\right\} = \frac{A^5}{y_0 x_{-1}^{\ 2}} \qquad y_{17} = \max\left\{\frac{A}{y_{15}},\frac{x_{16}}{y_{15}}\right\} = \max\left\{\frac{A^2}{x_0 y_{-1}},\frac{A^5}{x_0 y_{-1}^{\ 2}}\right\} = \frac{A}{x_0 y_{-1}}$$

$$x_{18} = \max\left\{\frac{A}{x_{16}},\frac{y_{17}}{x_{16}}\right\} = \max\left\{\frac{y_{-1}}{A^3},\frac{1}{x_0 A^3}\right\} = \frac{1}{x_0 A^3} \qquad y_{18} = \max\left\{\frac{A}{y_{16}},\frac{x_5}{y_4}\right\} = \max\left\{\frac{x_{-1}}{A^3},\frac{A}{y_0 x_{-1}}\right\} = \frac{A}{y_0 x_{-1}}$$

$$x_{19} = \max\left\{\frac{A}{x_{15}},\frac{y_{16}}{x_{15}}\right\} = \max\left\{\frac{y_0 x_{-1}^{\ 2}}{A^4},\frac{x_{-1}}{A^4}\right\} = \frac{x_{-1}}{A^4} \qquad y_{19} = \max\left\{\frac{A}{y_{15}},\frac{x_{16}}{y_{15}}\right\} = \max\left\{x_0 y_{-1},\frac{y_{-1}}{A^4}\right\} = \frac{y_{-1}}{A^4}$$

$$x_{20} = \max\left\{\frac{A}{x_{16}},\frac{y_{17}}{x_{16}}\right\} = \max\left\{A^4 x_0,\frac{x_0 y_{-1}}{A}\right\} = A^4 x_0 \qquad y_{20} = \max\left\{\frac{A}{y_{16}},\frac{x_5}{y_4}\right\} = \max\left\{y_0 x_{-1},\frac{y_0 x_{-1}^{\ 2}}{A^5}\right\} = \frac{y_0 x_{-1}^{\ 2}}{A^5}$$

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Teorem 4.7: Eğer A<1 ise $x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$ denkleminin $(x_n; y_n)$ çözümleri $x_{-1} < x_0 < y_{-1} < y_0 < A < 1, x_{-1} < y_0 < y_{-1} < x_0 < A < 1$ başlangıç şartlarına göre aşağıdaki şekildedir:

X_n ÇÖZÜMLERİ

$$x_{10n+1} = \frac{A^{2n+1}}{x_{-1}};$$

n=0,1,2 için $x_{10n+2} = \frac{A}{x_0 y_{-1}}$, $n \geq 3$ için $x_{10n+2} = \frac{A^{2n+2}}{x_0 y_{-1}^2}$;

n=0,1,2,3,4,5 için $x_{10n+3} = \frac{1}{A^{2n} y_0}$, $n \geq 6$ için $x_{10n+3} = \frac{A}{x_{-1} y_0}$;

$$x_{10n+4} = \frac{y_{-1}}{A^{2n+1}};$$

$$x_{10n+5} = A^{2n+1} \cdot y_0;$$

$$x_{10n+6} = \frac{A^{2n+2}}{y_{-1}};$$

$$x_{10n+7} = \frac{A}{x_{-1} y_0};$$

n=0,1,2 için $x_{10n+8} = \frac{1}{A^{2n+1} x_0}$, $n \geq 3$ için $x_{10n+8} = \frac{A}{x_0 y_{-1}}$;

$$x_{10n+9} = \frac{x_{-1}}{A^{2n+2}};$$

n=0,1 için $x_{10n+10} = A^{2n+2} \cdot x_0$, $n = 2$ için $x_{10n+10} = \frac{x_0 y_{-1}}{A}$, $n \geq 3$ için

$$x_{10n+10} = \frac{x_0 y_{-1}^2}{A^{2n+3}};$$

Y_n ÇÖZÜMLERİ

$$y_{10n+1} = \frac{A^{2n+1}}{y_{-1}};$$

n=0,1,2,3,4,5 için $y_{10n+2} = \frac{A}{x_{-1}y_0}$, $n \geq 6$ için $y_{10n+2} = \frac{A^{2n+2}}{x_{-1}^2 y_0}$;

n=0,1,2 için $y_{10n+3} = \frac{1}{A^{2n}x_0}$, $n \geq 3$ için $y_{10n+3} = \frac{A}{x_0 y_{-1}}$;

$$y_{10n+4} = \frac{x_{-1}}{A^{2n+1}};$$

n=0,1,2 için $y_{10n+5} = A^{2n+1} \cdot x_0$, $n \geq 3$ iç in $y_{10n+5} = \frac{x_0 y_{-1}^2}{A^{2n+2}}$;

$$y_{10n+6} = \frac{A^{2n+2}}{x_{-1}};$$

n=0,1,2 için $y_{10n+7} = \frac{A}{x_0 y_{-1}}$, $n \geq 3$ için $y_{10n+7} = \frac{A^{2n+3}}{x_0 y_{-1}^2}$;

$$y_{10n+8} = \frac{1}{A^{2n+1}y_0};$$

$$y_{10n+9} = \frac{y_{-1}}{A^{2n+2}};$$

$$y_{10n+10} = A^{2n+2} \cdot y_0;$$

İspat:

Bu teoremin ispatını n nin değerleri için gösterelim.

$$x_1 = \max\left\{\frac{A}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \frac{A}{x_{-1}} \qquad \mathrm{y}_1 = \max\left\{\frac{A}{y_{-1}}, \frac{x_0}{y_{-1}}\right\} = \frac{A}{y_{-1}}$$

$$x_2 = \max\left\{\frac{A}{x_0}, \frac{y_1}{x_0}\right\} = \max\left\{\frac{A}{x_0}, \frac{A}{y_{-1}x_0}\right\} = \frac{A}{y_{-1}x_0} \qquad \mathrm{y}_2 = \max\left\{\frac{A}{y_0}, \frac{x_1}{y_0}\right\} = \max\left\{\frac{A}{y_0}, \frac{A}{x_{-1}y_0}\right\} = \frac{A}{x_{-1}y_0}$$

$$x_3 = \max\left\{\frac{A}{x_0}, \frac{y_1}{x_0}\right\} = \max\left\{\mathrm{x}_{-1}, \frac{1}{y_0}\right\} = \frac{1}{y_0} \qquad \mathrm{y}_3 = \max\left\{\frac{A}{y_1}, \frac{x_2}{y_1}\right\} = \max\left\{\frac{Ay_{-1}}{A}, \frac{Ay_{-1}}{x_0 A y_{-1}}\right\} = \frac{1}{x_0}$$

$$x_4 = \max\left\{\frac{A}{x_2}, \frac{y_3}{x_2}\right\} = \max\left\{x_0 y_{-1}, \frac{y_{-1}}{A}\right\} = \frac{y_{-1}}{A} \qquad \mathrm{y}_4 = \max\left\{\frac{A}{y_2}, \frac{x_3}{y_2}\right\} = \max\left\{y_0 x_{-1}, \frac{x_{-1}}{A}\right\} = \frac{x_{-1}}{A}$$

$$x_5 = \max\left\{\frac{A}{x_3}, \frac{y_4}{x_3}\right\} = \max\left\{\mathrm{A}\, y_0, x_{-1}\frac{y_0}{A}\right\} = \mathrm{A}\, y_0 \qquad \mathrm{y}_5 = \max\left\{\frac{A}{y_3}, \frac{x_4}{y_3}\right\} = \max\left\{\mathrm{A}\, x_0, y_{-1}\frac{x_0}{A}\right\} = \mathrm{A}\, x_0$$

$$x_6 = \max\left\{\frac{A}{x_4}, \frac{y_5}{x_4}\right\} = \max\left\{\frac{A^2}{y_{-1}}, \frac{A^2 x_0}{y_{-1}}\right\} = \frac{A^2}{y_{-1}} \qquad \mathrm{y}_6 = \max\left\{\frac{A}{y_4}, \frac{x_5}{y_4}\right\} = \max\left\{\frac{A^2}{x_{-1}}, \frac{A^2 y_0}{x_{-1}}\right\} = \frac{A^2}{x_{-1}}$$

$$x_7 = \max\left\{\frac{A}{x_{-1}},\frac{y_0}{x_{-1}}\right\} = \left\{\frac{1}{y_0},\frac{A}{x_{-1}y_0}\right\} = \frac{A}{x_{-1}y_0} \qquad y_7 = \max\left\{\frac{A}{y_5},\frac{x_6}{y_5}\right\} = \left\{\frac{1}{x_0},\frac{A}{x_0y_{-1}}\right\} = \frac{A}{x_0y_{-1}}$$

$$x_8 = \max\left\{\frac{A}{x_6},\frac{y_7}{x_6}\right\} = \max\left\{\frac{y_{-1}}{A},\frac{1}{Ax_0}\right\} = \frac{1}{Ax_0} \qquad y_8 = \max\left\{\frac{A}{y_6},\frac{x_7}{y_6}\right\} = \max\left\{\frac{x_{-1}}{A},\frac{1}{Ay_0}\right\} = \frac{1}{Ay_0}$$

$$x_9 = \max\left\{\frac{A}{x_7},\frac{y_8}{x_7}\right\} = \max\left\{y_0x_{-1},\frac{x_{-1}}{A^2}\right\} = \frac{x_{-1}}{A^2} \qquad y_9 = \max\left\{\frac{A}{y_7},\frac{x_8}{y_7}\right\} = \left\{x_0y_{-1},\frac{y_{-1}}{A^2}\right\} = \frac{y_{-1}}{A^2}$$

$$x_{10} = \max\left\{\frac{A}{x_8},\frac{y_9}{x_8}\right\} = \max\left\{A^2x_0,\frac{Ax_0y_{-1}}{A^2}\right\} = A^2x_0 \qquad y_{10} = \max\left\{\frac{A}{y_8},\frac{x_9}{y_8}\right\} = \max\left\{A^2y_0,\frac{Ay_0x_{-1}}{A^2}\right\} = A^2y_0$$

$$x_{11} = \max\left\{\frac{A}{x_9},\frac{y_{10}}{x_9}\right\} = \max\left\{\frac{A^3}{x_{-1}},\frac{A^4y_0}{x_{-1}}\right\} = \frac{A^3}{x_{-1}} \qquad y_{11} = \max\left\{\frac{A}{y_9},\frac{x_{10}}{y_9}\right\} = \max\left\{\frac{A^3}{y_{-1}},\frac{A^4x_0}{y_{-1}}\right\} = \frac{A^3}{y_{-1}}$$

$$x_{12} = \max\left\{\frac{A}{x_{10}},\frac{y_{11}}{x_{10}}\right\} = \max\left\{\frac{1}{Ax_0},\frac{A}{x_0y_{-1}}\right\} = \frac{A}{x_0y_{-1}} \qquad y_{12} = \max\left\{\frac{A}{y_{10}},\frac{x_{11}}{y_{10}}\right\} = \max\left\{\frac{1}{Ay_0},\frac{A}{y_0x_{-1}}\right\} = \frac{A}{y_0x_{-1}}$$

$$x_{13} = \max\left\{\frac{A}{x_{11}},\frac{y_{12}}{x_{11}}\right\} = \left\{\frac{x_{-1}}{A^2},\frac{1}{A^2y_0}\right\} = \frac{1}{A^2y_0} \qquad y_{13} = \max\left\{\frac{A}{y_{11}},\frac{x_{12}}{y_{11}}\right\} = \left\{\frac{y_{-1}}{A^2},\frac{1}{A^2x_0}\right\} = \frac{1}{A^2x_0}$$

$$x_{14} = \max\left\{\frac{A}{x_{12}},\frac{y_{13}}{x_{12}}\right\} = \max\left\{x_0y_{-1},\frac{y_{-1}}{A^3}\right\} = \frac{y_{-1}}{A^3} \qquad y_{14} = \max\left\{\frac{A}{y_{12}},\frac{x_{13}}{y_{12}}\right\} = \max\left\{y_0x_{-1},\frac{x_{-1}}{A^3}\right\} = \frac{x_{-1}}{A^3}$$

$$x_{15} = \max\left\{\frac{A}{x_{13}},\frac{y_{14}}{x_{13}}\right\} = \max\left\{A^3y_0,\frac{y_0x_{-1}}{A}\right\} = A^3y_0 \qquad y_{15} = \max\left\{\frac{A}{y_{13}},\frac{x_{14}}{y_{13}}\right\} = \max\left\{A^3x_0,\frac{x_0y_{-1}}{A}\right\} = A^3x_0$$

$$x_{16} = \max\left\{\frac{A}{x_{14}},\frac{y_{15}}{x_{14}}\right\} = \max\left\{\frac{A^4}{y_{-1}},\frac{x_0A^6}{y_{-1}}\right\} = \frac{A^4}{y_{-1}} \qquad y_{16} = \max\left\{\frac{A}{y_{14}},\frac{x_{15}}{y_{14}}\right\} = \max\left\{\frac{A^4}{x_{-1}},\frac{A^6y_0}{x_{-1}}\right\} = \frac{A^4}{x_{-1}}$$

$$x_{17} = \max\left\{\frac{A}{x_{15}},\frac{y_{16}}{x_{15}}\right\} = \max\left\{\frac{1}{A^2y_0},\frac{A}{y_0x_{-1}}\right\} = \frac{A}{y_0x_{-1}} \qquad y_{17} = \max\left\{\frac{A}{y_{15}},\frac{x_{16}}{y_{15}}\right\} = \max\left\{\frac{1}{x_0A^2},\frac{A}{x_0y_{-1}}\right\} = \frac{A}{x_0y_{-1}}$$

$$x_{18} = \max\left\{\frac{A}{x_{16}},\frac{y_{17}}{x_{16}}\right\} = \max\left\{\frac{y_{-1}}{A^3},\frac{1}{x_0A^3}\right\} = \frac{1}{x_0A^3} \qquad y_{18} = \max\left\{\frac{A}{y_{16}},\frac{x_5}{y_4}\right\} = \max\left\{\frac{x_{-1}}{A^3},\frac{1}{y_0A^3}\right\} = \frac{1}{y_0A^3}$$

$$x_{19} = \max\left\{\frac{A}{x_{15}},\frac{y_{16}}{x_{15}}\right\} = \max\left\{y_0x_{-1},\frac{x_{-1}}{A^4}\right\} = \frac{x_{-1}}{A^4} \qquad y_{19} = \max\left\{\frac{A}{y_{15}},\frac{x_{16}}{y_{15}}\right\} = \max\left\{x_0y_{-1},\frac{y_{-1}}{A^4}\right\} = \frac{y_{-1}}{A^4}$$

$$x_{20} = \max\left\{\frac{A}{x_{16}},\frac{y_{17}}{x_{16}}\right\} = \max\left\{A^4x_0,\frac{x_0y_{-1}}{A}\right\} = A^4x_0 \qquad y_{20} = \max\left\{\frac{A}{y_{16}},\frac{x_5}{y_4}\right\} = \max\left\{A^4y_0,\frac{x_{-1}y_0}{A}\right\} = A^4y_0$$

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elde edilir.

Teorem 4.8:Eğer A<1 ise $x_{n+1}=\max\left\{\frac{A}{x_{n-1}},\frac{y_n}{x_{n-1}}\right\};y_{n+1}=\max\left\{\frac{A}{y_{n-1}},\frac{x_n}{y_{n-1}}\right\}$ denkleminin $(x_n;y_n)$ çözümleri $x_{-1}<x_0<y_0<y_{-1}<A<1, x_{-1}<y_0<x_0<y_{-1}<A<1$ başlangıç şartlarına göre aşağıdaki şekildedir. n=0,1,2,... için

X_n ÇÖZÜMLERİ

$$x_{10n+1}=\frac{A^{2n+1}}{x_{-1}};$$

n=0,1 için $x_{10n+2}=\frac{A}{x_0y_{-1}}$, $n\geq 2$ için $x_{10n+2}=\frac{A^{2n+2}}{x_0y_{-1}^2}$;

$$x_{10n+3}=\frac{1}{A^{2n}y_0};$$

$$x_{10n+4}=\frac{y_{-1}}{A^{2n+1}};$$

$$x_{10n+5}=A^{2n+1}\cdot y_0,$$

$$x_{10n+6}=\frac{A^{2n+2}}{y_{-1}};$$

$$x_{10n+7}=\frac{A}{x_{-1}y_0};$$

n=0 için $x_{10n+8}=\frac{1}{Ax_0}$, $n\geq 1$ için $x_{10n+8}=\frac{A}{x_0y_{-1}}$;

$$x_{10n+9}=\frac{x_{-1}}{A^{2n+2}};$$

n=0 için $x_{10n+10}=A^2\cdot x_0$, $n\geq 1$ için $x_{10n+10}=\frac{x_0y_{-1}^2}{A^{2n+3}}$;

Y_n ÇÖZÜMLERİ

$$y_{10n+1}=\frac{A^{2n+1}}{y_{-1}};$$

n=0,1,2,3,4,5 için $y_{10n+2}=\frac{A}{x_{-1}y_0}$, $n\geq 6$ için $y_{10n+2}=\frac{A^{2n+2}}{x_{-1}^2y_0}$;

n=0,1 için $y_{10n+3}=\dfrac{1}{A^{2n}x_0}$, $n\geq 2$ için $y_{10n+3}=\dfrac{A}{x_0y_{-1}}$;

$$y_{10n+4}=\frac{x_{-1}}{A^{2n+1}};$$

n=0 için $y_{10n+5}=Ax_0$, $n=1$ için $y_{10n+5}=\dfrac{x_0y_{-1}}{A}$, $n\geq 2$ için $y_{10n+5}=\dfrac{x_0y_{-1}^2}{A^{2n+2}}$;

$$y_{10n+6}=\frac{A^{2n+2}}{x_{-1}};$$

n=0 için $y_{10n+7}=\dfrac{A}{x_0y_{-1}}$, $n\geq 1$ için $y_{10n+7}=\dfrac{A^{2n+3}}{x_0y_{-1}^2}$;

$$y_{10n+8}=\frac{1}{A^{2n+1}y_0};$$

$$y_{10n+9}=\frac{y_{-1}}{A^{2n+2}};$$

$$y_{10n+10}=A^{2n+2}\cdot y_0;$$

İspat:

Bu teoremin ispatını n nin değerleri için gösterelim.

$$x_1=\max\left\{\frac{A}{x_{-1}},\frac{y_0}{x_{-1}}\right\}=\frac{A}{x_{-1}} \qquad y_1=\max\left\{\frac{A}{y_{-1}},\frac{x_0}{y_{-1}}\right\}=\frac{A}{y_{-1}}$$

$$x_2=\max\left\{\frac{A}{x_0},\frac{y_1}{x_0}\right\}=\max\left\{\frac{A}{x_0},\frac{A}{y_{-1}x_0}\right\}=\frac{A}{y_{-1}x_0} \qquad y_2=\max\left\{\frac{A}{y_0},\frac{x_1}{y_0}\right\}=\max\left\{\frac{A}{y_0},\frac{A}{x_{-1}y_0}\right\}=\frac{A}{x_{-1}y_0}$$

$$x_3=\max\left\{\frac{A}{x_0},\frac{y_1}{x_0}\right\}=\max\left\{x_{-1},\frac{1}{y_0}\right\}=\frac{1}{y_0} \qquad y_3=\max\left\{\frac{A}{y_1},\frac{x_2}{y_1}\right\}=\max\left\{\frac{Ay_{-1}}{A},\frac{Ay_{-1}}{x_0Ay_{-1}}\right\}=\frac{1}{x_0}$$

$$x_4=\max\left\{\frac{A}{x_2},\frac{y_3}{x_2}\right\}=\max\left\{x_0y_{-1},\frac{y_{-1}}{A}\right\}=\frac{y_{-1}}{A} \qquad y_4=\max\left\{\frac{A}{y_2},\frac{x_3}{y_2}\right\}=\max\left\{y_0x_{-1},\frac{x_{-1}}{A}\right\}=\frac{x_{-1}}{A}$$

$$x_5=\max\left\{\frac{A}{x_3},\frac{y_4}{x_3}\right\}=\max\left\{A\,y_0,x_{-1}\frac{y_0}{A}\right\}=A\,y_0 \qquad y_5=\max\left\{\frac{A}{y_3},\frac{x_4}{y_3}\right\}=\max\left\{A\,x_0,y_{-1}\frac{x_0}{A}\right\}=A\,x_0$$

$$x_6=\max\left\{\frac{A}{x_4},\frac{y_5}{x_4}\right\}=\max\left\{\frac{A^2}{y_{-1}},\frac{A^2x_0}{y_{-1}}\right\}=\frac{A^2}{y_{-1}} \qquad y_6=\max\left\{\frac{A}{y_4},\frac{x_5}{y_4}\right\}=\max\left\{\frac{A^2}{x_{-1}},\frac{A^2y_0}{x_{-1}}\right\}=\frac{A^2}{x_{-1}}$$

$$x_7 = \max\left\{\frac{A}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \left\{\frac{1}{y_0}, \frac{A}{x_{-1}y_0}\right\} = \frac{A}{x_{-1}y_0}$$

$$y_7 = \max\left\{\frac{A}{y_5}, \frac{x_6}{y_5}\right\} = \left\{\frac{1}{x_0}, \frac{A}{x_0y_{-1}}\right\} = \frac{A}{x_0y_{-1}}$$

$$x_8 = \max\left\{\frac{A}{x_6}, \frac{y_7}{x_6}\right\} = \max\left\{\frac{y_{-1}}{A}, \frac{1}{Ax_0}\right\} = \frac{1}{Ax_0}$$

$$y_8 = \max\left\{\frac{A}{y_6}, \frac{x_7}{y_6}\right\} = \max\left\{\frac{x_{-1}}{A}, \frac{1}{Ay_0}\right\} = \frac{1}{Ay_0}$$

$$x_9 = \max\left\{\frac{A}{x_7}, \frac{y_8}{x_7}\right\} = \max\left\{y_0x_{-1}, \frac{x_{-1}}{A^2}\right\} = \frac{x_{-1}}{A^2}$$

$$y_9 = \max\left\{\frac{A}{y_7}, \frac{x_8}{y_7}\right\} = \left\{x_0y_{-1}, \frac{y_{-1}}{A^2}\right\} = \frac{y_{-1}}{A^2}$$

$$x_{10} = \max\left\{\frac{A}{x_8}, \frac{y_9}{x_8}\right\} = \max\left\{A^2x_0, \frac{Ax_0y_{-1}}{A^2}\right\} = A^2x_0$$

$$y_{10} = \max\left\{\frac{A}{y_8}, \frac{x_9}{y_8}\right\} = \max\left\{A^2y_0, \frac{Ay_0x_{-1}}{A^2}\right\} = A^2y_0$$

$$x_{11} = \max\left\{\frac{A}{x_9}, \frac{y_{10}}{x_9}\right\} = \max\left\{\frac{A^3}{x_{-1}}, \frac{A^4y_0}{x_{-1}}\right\} = \frac{A^3}{x_{-1}}$$

$$y_{11} = \max\left\{\frac{A}{y_9}, \frac{x_{10}}{y_9}\right\} = \max\left\{\frac{A^3}{y_{-1}}, \frac{A^4x_0}{y_{-1}}\right\} = \frac{A^3}{y_{-1}}$$

$$x_{12} = \max\left\{\frac{A}{x_{10}}, \frac{y_{11}}{x_{10}}\right\} = \max\left\{\frac{1}{Ax_0}, \frac{A}{x_0y_{-1}}\right\} = \frac{A}{x_0y_{-1}}$$

$$y_{12} = \max\left\{\frac{A}{y_{10}}, \frac{x_{11}}{y_{10}}\right\} = \max\left\{\frac{1}{Ay_0}, \frac{A}{y_0x_{-1}}\right\} = \frac{A}{y_0x_{-1}}$$

$$x_{13} = \max\left\{\frac{A}{x_{11}}, \frac{y_{12}}{x_{11}}\right\} = \left\{\frac{x_{-1}}{A^2}, \frac{1}{A^2y_0}\right\} = \frac{1}{A^2y_0}$$

$$y_{13} = \max\left\{\frac{A}{y_{11}}, \frac{x_{12}}{y_{11}}\right\} = \left\{\frac{y_{-1}}{A^2}, \frac{1}{A^2x_0}\right\} = \frac{1}{A^2x_0}$$

$$x_{14} = \max\left\{\frac{A}{x_{12}}, \frac{y_{13}}{x_{12}}\right\} = \max\left\{x_0y_{-1}, \frac{y_{-1}}{A^3}\right\} = \frac{y_{-1}}{A^3}$$

$$y_{14} = \max\left\{\frac{A}{y_{12}}, \frac{x_{13}}{y_{12}}\right\} = \max\left\{y_0x_{-1}, \frac{x_{-1}}{A^3}\right\} = \frac{x_{-1}}{A^3}$$

$$x_{15} = \max\left\{\frac{A}{x_{13}}, \frac{y_{14}}{x_{13}}\right\} = \max\left\{A^3y_0, \frac{y_0x_{-1}}{A}\right\} = A^3y_0$$

$$y_{15} = \max\left\{\frac{A}{y_{13}}, \frac{x_{14}}{y_{13}}\right\} = \max\left\{A^3x_0, \frac{x_0y_{-1}}{A}\right\} = \frac{x_0y_{-1}}{A}$$

$$x_{16} = \max\left\{\frac{A}{x_{14}}, \frac{y_{15}}{x_{14}}\right\} = \max\left\{\frac{A^4}{y_{-1}}, x_0A^2\right\} = \frac{A^4}{y_{-1}}$$

$$y_{16} = \max\left\{\frac{A}{y_{14}}, \frac{x_{15}}{y_{14}}\right\} = \max\left\{\frac{A^4}{x_{-1}}, \frac{A^6y_0}{x_{-1}}\right\} = \frac{A^4}{x_{-1}}$$

$$x_{17} = \max\left\{\frac{A}{x_{15}}, \frac{y_{16}}{x_{15}}\right\} = \max\left\{\frac{1}{A^2y_0}, \frac{A}{y_0x_{-1}}\right\} = \frac{A}{y_0x_{-1}}$$

$$y_{17} = \max\left\{\frac{A}{y_{15}}, \frac{x_{16}}{y_{15}}\right\} = \max\left\{\frac{A^2}{x_0y_{-1}}, \frac{A^5}{x_0y_{-1}^{\ 2}}\right\} = \frac{A^5}{x_0y_{-1}^{\ 2}}$$

$$x_{18} = \max\left\{\frac{A}{x_{16}}, \frac{y_{17}}{x_{16}}\right\} = \max\left\{\frac{y_{-1}}{A^3}, \frac{A}{x_0y_{-1}}\right\} = \frac{A}{x_0y_{-1}}$$

$$y_{18} = \max\left\{\frac{A}{y_{16}}, \frac{x_5}{y_4}\right\} = \max\left\{\frac{x_{-1}}{A^3}, \frac{1}{y_0A^3}\right\} = \frac{1}{y_0A^3}$$

$$x_{19} = \max\left\{\frac{A}{x_{15}}, \frac{y_{16}}{x_{15}}\right\} = \max\left\{y_0x_{-1}, \frac{x_{-1}}{A^4}\right\} = \frac{x_{-1}}{A^4}$$

$$y_{19} = \max\left\{\frac{A}{y_{17}}, \frac{x_{18}}{y_{17}}\right\} = \max\left\{\frac{x_0y_{-1}^{\ 2}}{A^5}, \frac{y_{-1}}{A^4}\right\} = \frac{y_{-1}}{A^4}$$

$$x_{20} = \max\left\{\frac{A}{x_{16}}, \frac{y_{17}}{x_{16}}\right\} = \max\left\{x_0y_{-1}, \frac{x_0y_{-1}^{\ 2}}{A^5}\right\} = \frac{x_0y_{-1}^{\ 2}}{A^5}$$

$$y_{20} = \max\left\{\frac{A}{y_{16}}, \frac{x_5}{y_4}\right\} = \max\left\{A^4y_0, \frac{x_{-1}y_0}{A}\right\} = A^4y_0$$

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Teorem 4.9: Eğer A<1 ise $x_{n+1}=\max\left\{\frac{A}{x_{n-1}},\frac{y_n}{x_{n-1}}\right\};y_{n+1}=\max\left\{\frac{A}{y_{n-1}},\frac{x_n}{y_{n-1}}\right\}$ denkleminin

$(x_n;y_n)$ çözümleri $x_{-1}<y_{-1}<x_0<y_0<A<1, x_{-1}<y_{-1}<y_0<x_0<A<1$ başlangıç şartlarına göre aşağıdaki şekildedir:

X_n ÇÖZÜMLERİ

$$x_{10n+1}=\frac{A^{2n+1}}{x_{-1}};$$

$$x_{10n+2}=\frac{A}{x_0y_{-1}};$$

$$x_{10n+3}=\frac{1}{A^{2n}y_0};$$

$$x_{10n+4}=\frac{y_{-1}}{A^{2n+1}};$$

$$x_{10n+5}=A^{2n+1}\cdot y_0;$$

$$x_{10n+6}=\frac{A^{2n+2}}{y_{-1}};$$

$$x_{10n+7}=\frac{A}{x_{-1}y_0};$$

n=0,1,2,3,4 için $x_{10n+8}=\frac{1}{A^{2n+1}x_0}$, $n\geq 5$ için $x_{10n+8}=\frac{A}{x_0y_{-1}}$;

$$x_{10n+9}=\frac{x_{-1}}{A^{2n+2}};$$

n=0,1,2,3,4 için $x_{10n+10}=A^{2n+2}\cdot x_0$, $n\geq 5$ için $x_{10n+10}=\frac{x_0y_{-1}^2}{A^{13}}$;

Y_n ÇÖZÜMLERİ

$$y_{10n+1}=\frac{A^{2n+1}}{y_{-1}};$$

$$y_{10n+2}=\frac{A}{x_{-1}y_0};$$

$$y_{10n+3}=\frac{1}{A^{2n}x_0};$$

$$y_{10n+4}=\frac{x_{-1}}{A^{2n+1}};$$

n=0,1,2,3,4 için $y_{10n+5}=A^{2n+1}\cdot x_0$, $n\geq 5$ için $y_{10n+5}=\frac{x_0y_{-1}}{A}$;

$$y_{10n+6}=\frac{A^{2n+2}}{x_{-1}};$$

n=0,1,2,3,4 için $y_{10n+7}=\frac{A}{x_0y_{-1}}$, $n\geq 5$ için $y_{10n+7}=\frac{A^{2n+3}}{x_0y_{-1}^2}$;

$$y_{10n+8}=\frac{1}{A^{2n+1}y_0};$$

$$y_{10n+9}=\frac{y_{-1}}{A^{2n+2}};$$

$$y_{10n+10}=A^{2n+2}\cdot y_0;$$

İspat:

Bu teoremin ispatını n nin değerleri için gösterelim.

$$x_1=\max\left\{\frac{A}{x_{-1}},\frac{y_0}{x_{-1}}\right\}=\frac{A}{x_{-1}} \qquad y_1=\max\left\{\frac{A}{y_{-1}},\frac{x_0}{y_{-1}}\right\}=\frac{A}{y_{-1}}$$

$$x_2=\max\left\{\frac{A}{x_0},\frac{y_1}{x_0}\right\}=\max\left\{\frac{A}{x_0},\frac{A}{y_{-1}x_0}\right\}=\frac{A}{y_{-1}x_0} \qquad y_2=\max\left\{\frac{A}{y_0},\frac{x_1}{y_0}\right\}=\max\left\{\frac{A}{y_0},\frac{A}{x_{-1}y_0}\right\}=\frac{A}{x_{-1}y_0}$$

$$x_3=\max\left\{\frac{A}{x_0},\frac{y_1}{x_0}\right\}=\max\left\{x_{-1},\frac{1}{y_0}\right\}=\frac{1}{y_0} \qquad y_3=\max\left\{\frac{A}{y_1},\frac{x_2}{y_1}\right\}=\max\left\{\frac{Ay_{-1}}{A},\frac{Ay_{-1}}{x_0Ay_{-1}}\right\}=\frac{1}{x_0}$$

$$x_4=\max\left\{\frac{A}{x_2},\frac{y_3}{x_2}\right\}=\max\left\{x_0y_{-1},\frac{y_{-1}}{A}\right\}=\frac{y_{-1}}{A} \qquad y_4=\max\left\{\frac{A}{y_2},\frac{x_3}{y_2}\right\}=\max\left\{y_0x_{-1},\frac{x_{-1}}{A}\right\}=\frac{x_{-1}}{A}$$

$$x_5 = \max\left\{\frac{A}{x_3}, \frac{y_4}{x_3}\right\} = \max\left\{\mathrm{A}\, y_0, x_{-1}\frac{y_0}{A}\right\} = \mathrm{A}\, y_0$$

$$x_6 = \max\left\{\frac{A}{x_4}, \frac{y_5}{x_4}\right\} = \max\left\{\frac{A^2}{y_{-1}}, \frac{A^2 x_0}{y_{-1}}\right\} = \frac{A^2}{y_{-1}}$$

$$x_7 = \max\left\{\frac{A}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \left\{\frac{1}{y_0}, \frac{A}{x_{-1}y_0}\right\} = \frac{A}{x_{-1}y_0}$$

$$x_8 = \max\left\{\frac{A}{x_6}, \frac{y_7}{x_6}\right\} = \max\left\{\frac{y_{-1}}{A}, \frac{1}{Ax_0}\right\} = \frac{1}{Ax_0}$$

$$x_9 = \max\left\{\frac{A}{x_7}, \frac{y_8}{x_7}\right\} = \max\left\{\mathrm{y}_0\mathrm{x}_{-1}, \frac{x_{-1}}{A^2}\right\} = \frac{x_{-1}}{A^2}$$

$$x_{10} = \max\left\{\frac{A}{x_8}, \frac{y_9}{x_8}\right\} = \max\left\{A^2 x_0, \frac{Ax_0 y_{-1}}{A^2}\right\} = A^2 x_0$$

$$x_{11} = \max\left\{\frac{A}{x_9}, \frac{y_{10}}{x_9}\right\} = \max\left\{\frac{A^3}{x_{-1}}, \frac{A^4 y_0}{x_{-1}}\right\} = \frac{A^3}{x_{-1}}$$

$$x_{12} = \max\left\{\frac{A}{x_{10}}, \frac{y_{11}}{x_{10}}\right\} = \max\left\{\frac{1}{Ax_0}, \frac{A}{x_0 y_{-1}}\right\} = \frac{A}{x_0 y_{-1}}$$

$$x_{13} = \max\left\{\frac{A}{x_{11}}, \frac{y_{12}}{x_{11}}\right\} = \left\{\frac{x_{-1}}{A^2}, \frac{1}{A^2 y_0}\right\} = \frac{1}{A^2 y_0}$$

$$x_{14} = \max\left\{\frac{A}{x_{12}}, \frac{y_{13}}{x_{12}}\right\} = \max\left\{\mathrm{x}_0\mathrm{y}_{-1}, \frac{y_{-1}}{A^3}\right\} = \frac{y_{-1}}{A^3}$$

$$x_{15} = \max\left\{\frac{A}{x_{13}}, \frac{y_{14}}{x_{13}}\right\} = \max\left\{A^3\, \mathrm{y}_0, \frac{y_0 x_{-1}}{A}\right\} = A^3\, \mathrm{y}_0$$

$$x_{16} = \max\left\{\frac{A}{x_{14}}, \frac{y_{15}}{x_{14}}\right\} = \max\left\{\frac{A^4}{y_{-1}}, \frac{\mathrm{x}_0\, A^6}{y_{-1}}\right\} = \frac{A^4}{y_{-1}}$$

$$x_{17} = \max\left\{\frac{A}{x_{15}}, \frac{y_{16}}{x_{15}}\right\} = \max\left\{\frac{1}{A^2 y_0}, \frac{A}{y_0 x_{-1}}\right\} = \frac{A}{y_0 x_{-1}}$$

$$x_{18} = \max\left\{\frac{A}{x_{16}}, \frac{y_{17}}{x_{16}}\right\} = \max\left\{\frac{y_{-1}}{A^3}, \frac{1}{x_0 A^3}\right\} = \frac{1}{x_0 A^3}$$

$$\mathrm{y}_5 = \max\left\{\frac{A}{y_3}, \frac{x_4}{y_3}\right\} = \max\left\{\mathrm{A}\, x_0, y_{-1}\frac{x_0}{A}\right\} = \mathrm{A}\, x_0$$

$$\mathrm{y}_6 = \max\left\{\frac{A}{y_4}, \frac{x_5}{y_4}\right\} = \max\left\{\frac{A^2}{x_{-1}}, \frac{A^2 y_0}{x_{-1}}\right\} = \frac{A^2}{x_{-1}}$$

$$\mathrm{y}_7 = \max\left\{\frac{A}{y_5}, \frac{x_6}{y_5}\right\} = \left\{\frac{1}{x_0}, \frac{A}{x_0 y_{-1}}\right\} = \frac{A}{x_0 y_{-1}}$$

$$\mathrm{y}_8 = \max\left\{\frac{A}{y_6}, \frac{x_7}{y_6}\right\} = \max\left\{\frac{x_{-1}}{A}, \frac{1}{Ay_0}\right\} = \frac{1}{Ay_0}$$

$$\mathrm{y}_9 = \max\left\{\frac{A}{y_7}, \frac{x_8}{y_7}\right\} = \left\{\mathrm{x}_0\mathrm{y}_{-1}, \frac{y_{-1}}{A^2}\right\} = \frac{y_{-1}}{A^2}$$

$$\mathrm{y}_{10} = \max\left\{\frac{A}{y_8}, \frac{x_9}{y_8}\right\} = \max\left\{A^2 y_0, \frac{Ay_0 x_{-1}}{A^2}\right\} = A^2 y_0$$

$$\mathrm{y}_{11} = \max\left\{\frac{A}{y_9}, \frac{x_{10}}{y_9}\right\} = \max\left\{\frac{A^3}{y_{-1}}, \frac{A^4 x_0}{y_{-1}}\right\} = \frac{A^3}{y_{-1}}$$

$$\mathrm{y}_{12} = \max\left\{\frac{A}{y_{10}}, \frac{x_{11}}{y_{10}}\right\} = \max\left\{\frac{1}{Ay_0}, \frac{A}{y_0 x_{-1}}\right\} = \frac{A}{y_0 x_{-1}}$$

$$\mathrm{y}_{13} = \max\left\{\frac{A}{y_{11}}, \frac{x_{12}}{y_{11}}\right\} = \left\{\frac{y_{-1}}{A^2}, \frac{1}{A^2 x_0}\right\} = \frac{1}{A^2 x_0}$$

$$\mathrm{y}_{14} = \max\left\{\frac{A}{y_{12}}, \frac{x_{13}}{y_{12}}\right\} = \max\left\{\mathrm{y}_0\mathrm{x}_{-1}, \frac{x_{-1}}{A^3}\right\} = \frac{x_{-1}}{A^3}$$

$$\mathrm{y}_{15} = \max\left\{\frac{A}{y_{13}}, \frac{x_{14}}{y_{13}}\right\} = \max\left\{\mathrm{A}^3\mathrm{x}_0, \frac{x_0 y_{-1}}{A}\right\} = A^3 x_0$$

$$\mathrm{y}_{16} = \max\left\{\frac{A}{y_{14}}, \frac{x_{15}}{y_{14}}\right\} = \max\left\{\frac{A^4}{x_{-1}}, \frac{A^6 y_0}{x_{-1}}\right\} = \frac{A^4}{x_{-1}}$$

$$\mathrm{y}_{17} = \max\left\{\frac{A}{y_{15}}, \frac{x_{16}}{y_{15}}\right\} = \max\left\{\frac{1}{x_0 A^2}, \frac{A}{x_0 y_{-1}}\right\} = \frac{A}{x_0 y_{-1}}$$

$$\mathrm{y}_{18} = \max\left\{\frac{A}{y_{16}}, \frac{x_5}{y_4}\right\} = \max\left\{\frac{x_{-1}}{A^3}, \frac{1}{y_0 A^3}\right\} = \frac{1}{y_0 A^3}$$

$$x_{19} = \max\left\{\frac{A}{x_{15}}, \frac{y_{16}}{x_{15}}\right\} = \max\left\{y_0 \, x_{-1}, \frac{x_{-1}}{A^4}\right\} = \frac{x_{-1}}{A^4} \qquad y_{19} = \max\left\{\frac{A}{y_{15}}, \frac{x_{16}}{y_{15}}\right\} = \max\left\{x_0 y_{-1}, \frac{y_{-1}}{A^4}\right\} = \frac{y_{-1}}{A^4}$$

$$x_{20} = \max\left\{\frac{A}{x_{16}}, \frac{y_{17}}{x_{16}}\right\} = \max\left\{A^4 x_0, \frac{x_0 y_{-1}}{A}\right\} = A^4 x_0 \qquad y_{20} = \max\left\{\frac{A}{y_{16}}, \frac{x_5}{y_4}\right\} = \max\left\{A^4 y_0, \frac{x_{-1} y_0}{A}\right\} = A^4 y_0$$

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Teorem 4.10:Eğer A<1 ise $x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$ denkleminin $(x_n; y_n)$ çözümleri $y_{-1} < y_0 < x_{-1} < x_0 < A < 1$, $y_{-1} < x_0 < x_{-1} < y_0 < A < 1$ başlangıç şartlarına göre aşağıdaki şekildedir:

X_n ÇÖZÜMLERİ

$$x_{10n+1} = \frac{A^{2n+1}}{x_{-1}};$$

$$x_{10n+2} = \frac{A}{x_0 y_{-1}};$$

n=0,1,2 için $x_{10n+3} = \frac{1}{A^{2n} y_0}$, $n \geq 3$ için $x_{10n+3} = \frac{A}{x_{-1} y_0}$;

$$x_{10n+4} = \frac{y_{-1}}{A^{2n+1}};$$

n=0,1,2 için $x_{10n+5} = A^{2n+1} \cdot y_0$, $n \geq 3$ için $x_{10n+5} = \frac{x_{-1}^2 y_0}{A^{2n+2}}$;

$$x_{10n+6} = \frac{A^{2n+2}}{y_{-1}};$$

n=0,1,2 için $x_{10n+7} = \frac{A}{x_{-1} y_0}$, $n \geq 3$ için $x_{10n+7} = \frac{A^{2n+3}}{x_{-1}^2 y_0}$;

$$x_{10n+8} = \frac{1}{A^{2n+1} x_0};$$

$$x_{10n+9}=\frac{x_{-1}}{A^{2n+2}};$$

$$x_{10n+10}=A^{2n+2}\cdot x_0;$$

Y_n ÇÖZÜMLERİ

$$y_{10n+1}=\frac{A^{2n+1}}{y_{-1}};$$

n=0,1,2 için $y_{10n+2}=\frac{A}{x_{-1}y_0}$, $n\geq 3$ için $y_{10n+2}=\frac{A^{2n+2}}{x_{-1}^2y_0}$;

$$y_{10n+3}=\frac{1}{A^{2n}x_0};$$

$$y_{10n+4}=\frac{x_{-1}}{A^{2n+1}};$$

$$y_{10n+5}=A^{2n+1}\cdot x_0;$$

$$y_{10n+6}=\frac{A^{2n+2}}{x_{-1}};$$

$$y_{10n+7}=\frac{A}{x_0y_{-1}};$$

n=0,1,2 için $y_{10n+8}=\frac{1}{A^{2n+1}y_0}$, $n\geq 3$ için $y_{10n+8}=\frac{A}{x_{-1}y_0}$;

$$y_{10n+9}=\frac{y_{-1}}{A^{2n+2}};$$

n=0,1 için $y_{10n+10}=A^{2n+2}\cdot y_0$, n=2 için $y_{10n+10}=\frac{x_{-1}y_0}{A}$, $n\geq 3$ için

$$y_{10n+10}=\frac{x_{-1}^2y_0}{A^{2n+3}};$$

İspat:

Bu teoremin ispatını n nin değerleri için gösterelim.

$x_1=\max\left\{\frac{A}{x_{-1}},\frac{y_0}{x_{-1}}\right\}=\frac{A}{x_{-1}}$ $\qquad$ $y_1=\max\left\{\frac{A}{y_{-1}},\frac{x_0}{y_{-1}}\right\}=\frac{A}{y_{-1}}$

$$x_2 = \max\left\{\frac{A}{x_0},\frac{y_1}{x_0}\right\} = \max\left\{\frac{A}{x_0},\frac{A}{y_{-1}x_0}\right\} = \frac{A}{y_{-1}x_0}$$

$$y_2 = \max\left\{\frac{A}{y_0},\frac{x_1}{y_0}\right\} = \max\left\{\frac{A}{y_0},\frac{A}{x_{-1}y_0}\right\} = \frac{A}{x_{-1}y_0}$$

$$x_3 = \max\left\{\frac{A}{x_0},\frac{y_1}{x_0}\right\} = \max\left\{x_{-1},\frac{1}{y_0}\right\} = \frac{1}{y_0}$$

$$y_3 = \max\left\{\frac{A}{y_1},\frac{x_2}{y_1}\right\} = \max\left\{\frac{Ay_{-1}}{A},\frac{Ay_{-1}}{x_0Ay_{-1}}\right\} = \frac{1}{x_0}$$

$$x_4 = \max\left\{\frac{A}{x_2},\frac{y_3}{x_2}\right\} = \max\left\{x_0y_{-1},\frac{y_{-1}}{A}\right\} = \frac{y_{-1}}{A}$$

$$y_4 = \max\left\{\frac{A}{y_2},\frac{x_3}{y_2}\right\} = \max\left\{y_0x_{-1},\frac{x_{-1}}{A}\right\} = \frac{x_{-1}}{A}$$

$$x_5 = \max\left\{\frac{A}{x_3},\frac{y_4}{x_3}\right\} = \max\left\{A\,y_0, x_{-1}\frac{y_0}{A}\right\} = A\,y_0$$

$$y_5 = \max\left\{\frac{A}{y_3},\frac{x_4}{y_3}\right\} = \max\left\{A\,x_0, y_{-1}\frac{x_0}{A}\right\} = A\,x_0$$

$$x_6 = \max\left\{\frac{A}{x_4},\frac{y_5}{x_4}\right\} = \max\left\{\frac{A^2}{y_{-1}},\frac{A^2x_0}{y_{-1}}\right\} = \frac{A^2}{y_{-1}}$$

$$y_6 = \max\left\{\frac{A}{y_4},\frac{x_5}{y_4}\right\} = \max\left\{\frac{A^2}{x_{-1}},\frac{A^2y_0}{x_{-1}}\right\} = \frac{A^2}{x_{-1}}$$

$$x_7 = \max\left\{\frac{A}{x_{-1}},\frac{y_0}{x_{-1}}\right\} = \left\{\frac{1}{y_0},\frac{A}{x_{-1}y_0}\right\} = \frac{A}{x_{-1}y_0}$$

$$y_7 = \max\left\{\frac{A}{y_5},\frac{x_6}{y_5}\right\} = \left\{\frac{1}{x_0},\frac{A}{x_0y_{-1}}\right\} = \frac{A}{x_0y_{-1}}$$

$$x_8 = \max\left\{\frac{A}{x_6},\frac{y_7}{x_6}\right\} = \max\left\{\frac{y_{-1}}{A},\frac{1}{Ax_0}\right\} = \frac{1}{Ax_0}$$

$$y_8 = \max\left\{\frac{A}{y_6},\frac{x_7}{y_6}\right\} = \max\left\{\frac{x_{-1}}{A},\frac{1}{Ay_0}\right\} = \frac{1}{Ay_0}$$

$$x_9 = \max\left\{\frac{A}{x_7},\frac{y_8}{x_7}\right\} = \max\left\{y_0x_{-1},\frac{x_{-1}}{A^2}\right\} = \frac{x_{-1}}{A^2}$$

$$y_9 = \max\left\{\frac{A}{y_7},\frac{x_8}{y_7}\right\} = \left\{x_0y_{-1},\frac{y_{-1}}{A^2}\right\} = \frac{y_{-1}}{A^2}$$

$$x_{10} = \max\left\{\frac{A}{x_8},\frac{y_9}{x_8}\right\} = \max\left\{A^2x_0,\frac{Ax_0y_{-1}}{A^2}\right\} = A^2x_0$$

$$y_{10} = \max\left\{\frac{A}{y_8},\frac{x_9}{y_8}\right\} = \max\left\{A^2y_0,\frac{Ay_0x_{-1}}{A^2}\right\} = A^2y_0$$

$$x_{11} = \max\left\{\frac{A}{x_9},\frac{y_{10}}{x_9}\right\} = \max\left\{\frac{A^3}{x_{-1}},\frac{A^4y_0}{x_{-1}}\right\} = \frac{A^3}{x_{-1}}$$

$$y_{11} = \max\left\{\frac{A}{y_9},\frac{x_{10}}{y_9}\right\} = \max\left\{\frac{A^3}{y_{-1}},\frac{A^4x_0}{y_{-1}}\right\} = \frac{A^3}{y_{-1}}$$

$$x_{12} = \max\left\{\frac{A}{x_{10}},\frac{y_{11}}{x_{10}}\right\} = \max\left\{\frac{1}{Ax_0},\frac{A}{x_0y_{-1}}\right\} = \frac{A}{x_0y_{-1}}$$

$$y_{12} = \max\left\{\frac{A}{y_{10}},\frac{x_{11}}{y_{10}}\right\} = \max\left\{\frac{1}{Ay_0},\frac{A}{y_0x_{-1}}\right\} = \frac{A}{y_0x_{-1}}$$

$$x_{13} = \max\left\{\frac{A}{x_{11}},\frac{y_{12}}{x_{11}}\right\} = \left\{\frac{x_{-1}}{A^2},\frac{1}{A^2y_0}\right\} = \frac{1}{A^2y_0}$$

$$y_{13} = \max\left\{\frac{A}{y_{11}},\frac{x_{12}}{y_{11}}\right\} = \left\{\frac{y_{-1}}{A^2},\frac{1}{A^2x_0}\right\} = \frac{1}{A^2x_0}$$

$$x_{14} = \max\left\{\frac{A}{x_{12}},\frac{y_{13}}{x_{12}}\right\} = \max\left\{x_0y_{-1},\frac{y_{-1}}{A^3}\right\} = \frac{y_{-1}}{A^3}$$

$$y_{14} = \max\left\{\frac{A}{y_{12}},\frac{x_{13}}{y_{12}}\right\} = \max\left\{y_0x_{-1},\frac{x_{-1}}{A^3}\right\} = \frac{x_{-1}}{A^3}$$

$$x_{15} = \max\left\{\frac{A}{x_{13}},\frac{y_{14}}{x_{13}}\right\} = \max\left\{A^3y_0,\frac{y_0x_{-1}}{A}\right\} = A^3y_0$$

$$y_{15} = \max\left\{\frac{A}{y_{13}},\frac{x_{14}}{y_{13}}\right\} = \max\left\{A^3x_0,\frac{x_0y_{-1}}{A}\right\} = A^3x_0$$

$$x_{16} = \max\left\{\frac{A}{x_{14}},\frac{y_{15}}{x_{14}}\right\} = \max\left\{\frac{A^4}{y_{-1}},\frac{x_0A^6}{y_{-1}}\right\} = \frac{A^4}{y_{-1}}$$

$$y_{16} = \max\left\{\frac{A}{y_{14}},\frac{x_{15}}{y_{14}}\right\} = \max\left\{\frac{A^4}{x_{-1}},\frac{A^6y_0}{x_{-1}}\right\} = \frac{A^4}{x_{-1}}$$

$$x_{17} = \max\left\{\frac{A}{x_{15}}, \frac{y_{16}}{x_{15}}\right\} = \max\left\{\frac{1}{A^2 y_0}, \frac{A}{y_0 x_{-1}}\right\} = \frac{A}{y_0 x_{-1}}$$

$$y_{17} = \max\left\{\frac{A}{y_{15}}, \frac{x_{16}}{y_{15}}\right\} = \max\left\{\frac{1}{x_0 A^2}, \frac{A}{x_0 y_{-1}}\right\} = \frac{A}{x_0 y_{-1}}$$

$$x_{18} = \max\left\{\frac{A}{x_{16}}, \frac{y_{17}}{x_{16}}\right\} = \max\left\{\frac{y_{-1}}{A^3}, \frac{1}{x_0 A^3}\right\} = \frac{1}{x_0 A^3}$$

$$y_{18} = \max\left\{\frac{A}{y_{16}}, \frac{x_5}{y_4}\right\} = \max\left\{\frac{x_{-1}}{A^3}, \frac{1}{y_0 A^3}\right\} = \frac{1}{y_0 A^3}$$

$$x_{19} = \max\left\{\frac{A}{x_{15}}, \frac{y_{16}}{x_{15}}\right\} = \max\left\{y_0\, x_{-1}, \frac{x_{-1}}{A^4}\right\} = \frac{x_{-1}}{A^4}$$

$$y_{19} = \max\left\{\frac{A}{y_{15}}, \frac{x_{16}}{y_{15}}\right\} = \max\left\{x_0 y_{-1}, \frac{y_{-1}}{A^4}\right\} = \frac{y_{-1}}{A^4}$$

$$x_{20} = \max\left\{\frac{A}{x_{16}}, \frac{y_{17}}{x_{16}}\right\} = \max\left\{A^4 x_0, \frac{x_0 y_{-1}}{A}\right\} = A^4 x_0$$

$$y_{20} = \max\left\{\frac{A}{y_{16}}, \frac{x_5}{y_4}\right\} = \max\left\{A^4 y_0, \frac{x_{-1} y_0}{A}\right\} = A^4 y_0$$

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Teorem 4.11: Eğer A<1 ise $x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$

denkleminin $(x_n; y_n)$ çözümleri $y_{-1} < y_0 < x_0 < x_{-1} < A < 1$,

$y_{-1} < x_0 < y_0 < x_{-1} < A < 1$ başlangıç şartlarına göre aşağıdaki şekildedir.

X_n ÇÖZÜMLERİ

$$x_{10n+1} = \frac{A^{2n+1}}{x_{-1}};$$

$$x_{10n+2} = \frac{A}{x_0 y_{-1}};$$

n=0,1 için $x_{10n+3} = \frac{1}{A^{2n} y_0}$, $n \geq 2$ için $x_{10n+3} = \frac{A}{x_{-1} y_0}$;

$$x_{10n+4} = \frac{y_{-1}}{A^{2n+1}};$$

n=0 için $x_{10n+5} = A \cdot y_0$, n=1 için $x_{10n+5} = \frac{x_{-1} y_0}{A}$, $n \geq 2$ için $x_{10n+5} = \frac{x_{-1}^2 y_0}{A^{2n+2}}$;

$$x_{10n+6} = \frac{A^{2n+2}}{y_{-1}};$$

n=0 için $x_{10n+7} = \frac{A}{x_{-1}y_0}$, $n \geq 1$ için $x_{10n+7} = \frac{A^{2n+3}}{x_{-1}^2 y_0}$;

$$x_{10n+8} = \frac{1}{A^{2n+1}x_0};$$

$$x_{10n+9} = \frac{x_{-1}}{A^{2n+2}};$$

$$x_{10n+10} = A^{2n+2} \cdot x_0,$$

Y_n ÇÖZÜMLERİ

$$y_{10n+1} = \frac{A^{2n+1}}{y_{-1}};$$

n=0,1 için $y_{10n+2} = \frac{A}{x_{-1}y_0}$, $n \geq 2$ için $y_{10n+2} = \frac{A^{2n+2}}{x_{-1}^2 y_0}$;

$$y_{10n+3} = \frac{1}{A^{2n}x_0};$$

$$y_{10n+4} = \frac{x_{-1}}{A^{2n+1}};$$

$$y_{10n+5} = A^{2n+1} \cdot x_0;$$

$$y_{10n+6} = \frac{A^{2n+2}}{x_{-1}};$$

$$y_{10n+7} = \frac{A}{x_0 y_{-1}};$$

n=0 için $y_{10n+8} = \frac{1}{Ay_0}$, $n \geq 1$ için $y_{10n+8} = \frac{A}{x_{-1}y_0}$;

$$y_{10n+9} = \frac{y_{-1}}{A^{2n+2}};$$

n=0 için $y_{10n+10} = A^2 \cdot y_0$, $n \geq 1$ için $y_{10n+10} = \frac{x_{-1}^2 y_0}{A^{2n+3}}$;

İspat:

Bu teoremin ispatını n nin değerleri için gösterelim.

$$x_1=\max\left\{\frac{A}{x_{-1}},\frac{y_0}{x_{-1}}\right\}=\frac{A}{x_{-1}}$$

$$y_1=\max\left\{\frac{A}{y_{-1}},\frac{x_0}{y_{-1}}\right\}=\frac{A}{y_{-1}}$$

$$x_2=\max\left\{\frac{A}{x_0},\frac{y_1}{x_0}\right\}=\max\left\{\frac{A}{x_0},\frac{A}{y_{-1}x_0}\right\}=\frac{A}{y_{-1}x_0}$$

$$y_2=\max\left\{\frac{A}{y_0},\frac{x_1}{y_0}\right\}=\max\left\{\frac{A}{y_0},\frac{A}{x_{-1}y_0}\right\}=\frac{A}{x_{-1}y_0}$$

$$x_3=\max\left\{\frac{A}{x_0},\frac{y_1}{x_0}\right\}=\max\left\{x_{-1},\frac{1}{y_0}\right\}=\frac{1}{y_0}$$

$$y_3=\max\left\{\frac{A}{y_1},\frac{x_2}{y_1}\right\}=\max\left\{\frac{Ay_{-1}}{A},\frac{Ay_{-1}}{x_0Ay_{-1}}\right\}=\frac{1}{x_0}$$

$$x_4=\max\left\{\frac{A}{x_2},\frac{y_3}{x_2}\right\}=\max\left\{x_0y_{-1},\frac{y_{-1}}{A}\right\}=\frac{y_{-1}}{A}$$

$$y_4=\max\left\{\frac{A}{y_2},\frac{x_3}{y_2}\right\}=\max\left\{y_0x_{-1},\frac{x_{-1}}{A}\right\}=\frac{x_{-1}}{A}$$

$$x_5=\max\left\{\frac{A}{x_3},\frac{y_4}{x_3}\right\}=\max\left\{A\,y_0,x_{-1}\frac{y_0}{A}\right\}=A\,y_0$$

$$y_5=\max\left\{\frac{A}{y_3},\frac{x_4}{y_3}\right\}=\max\left\{A\,x_0,y_{-1}\frac{x_0}{A}\right\}=A\,x_0$$

$$x_6=\max\left\{\frac{A}{x_4},\frac{y_5}{x_4}\right\}=\max\left\{\frac{A^2}{y_{-1}},\frac{A^2x_0}{y_{-1}}\right\}=\frac{A^2}{y_{-1}}$$

$$y_6=\max\left\{\frac{A}{y_4},\frac{x_5}{y_4}\right\}=\max\left\{\frac{A^2}{x_{-1}},\frac{A^2y_0}{x_{-1}}\right\}=\frac{A^2}{x_{-1}}$$

$$x_7=\max\left\{\frac{A}{x_{-1}},\frac{y_0}{x_{-1}}\right\}=\left\{\frac{1}{y_0},\frac{A}{x_{-1}y_0}\right\}=\frac{A}{x_{-1}y_0}$$

$$y_7=\max\left\{\frac{A}{y_5},\frac{x_6}{y_5}\right\}=\left\{\frac{1}{x_0},\frac{A}{x_0y_{-1}}\right\}=\frac{A}{x_0y_{-1}}$$

$$x_8=\max\left\{\frac{A}{x_6},\frac{y_7}{x_6}\right\}=\max\left\{\frac{y_{-1}}{A},\frac{1}{Ax_0}\right\}=\frac{1}{Ax_0}$$

$$y_8=\max\left\{\frac{A}{y_6},\frac{x_7}{y_6}\right\}=\max\left\{\frac{x_{-1}}{A},\frac{1}{Ay_0}\right\}=\frac{1}{Ay_0}$$

$$x_9=\max\left\{\frac{A}{x_7},\frac{y_8}{x_7}\right\}=\max\left\{y_0x_{-1},\frac{x_{-1}}{A^2}\right\}=\frac{x_{-1}}{A^2}$$

$$y_9=\max\left\{\frac{A}{y_7},\frac{x_8}{y_7}\right\}=\left\{x_0y_{-1},\frac{y_{-1}}{A^2}\right\}=\frac{y_{-1}}{A^2}$$

$$x_{10}=\max\left\{\frac{A}{x_8},\frac{y_9}{x_8}\right\}=\max\left\{A^2x_0,\frac{Ax_0y_{-1}}{A^2}\right\}=A^2x_0$$

$$y_{10}=\max\left\{\frac{A}{y_8},\frac{x_9}{y_8}\right\}=\max\left\{A^2y_0,\frac{Ay_0x_{-1}}{A^2}\right\}=A^2y_0$$

$$x_{11}=\max\left\{\frac{A}{x_9},\frac{y_{10}}{x_9}\right\}=\max\left\{\frac{A^3}{x_{-1}},\frac{A^4y_0}{x_{-1}}\right\}=\frac{A^3}{x_{-1}}$$

$$y_{11}=\max\left\{\frac{A}{y_9},\frac{x_{10}}{y_9}\right\}=\max\left\{\frac{A^3}{y_{-1}},\frac{A^4x_0}{y_{-1}}\right\}=\frac{A^3}{y_{-1}}$$

$$x_{12}=\max\left\{\frac{A}{x_{10}},\frac{y_{11}}{x_{10}}\right\}=\max\left\{\frac{1}{Ax_0},\frac{A}{x_0y_{-1}}\right\}=\frac{A}{x_0y_{-1}}$$

$$y_{12}=\max\left\{\frac{A}{y_{10}},\frac{x_{11}}{y_{10}}\right\}=\max\left\{\frac{1}{A^2y_0},\frac{A}{y_0x_{-1}}\right\}=\frac{A}{y_0x_{-1}}$$

$$x_{13}=\max\left\{\frac{A}{x_{11}},\frac{y_{12}}{x_{11}}\right\}=\left\{\frac{x_{-1}}{A^2},\frac{1}{A^2y_0}\right\}=\frac{1}{A^2y_0}$$

$$y_{13}=\max\left\{\frac{A}{y_{11}},\frac{x_{12}}{y_{11}}\right\}=\left\{\frac{y_{-1}}{A^2},\frac{1}{A^2x_0}\right\}=\frac{1}{A^2x_0}$$

$$x_{14}=\max\left\{\frac{A}{x_{12}},\frac{y_{13}}{x_{12}}\right\}=\max\left\{x_0y_{-1},\frac{y_{-1}}{A^3}\right\}=\frac{y_{-1}}{A^3}$$

$$y_{14}=\max\left\{\frac{A}{y_{12}},\frac{x_{13}}{y_{12}}\right\}=\max\left\{y_0x_{-1},\frac{x_{-1}}{A^3}\right\}=\frac{x_{-1}}{A^3}$$

$$x_{15}=\max\left\{\frac{A}{x_{13}},\frac{y_{14}}{x_{13}}\right\}=\max\left\{A^3y_0,\frac{y_0x_{-1}}{A}\right\}=\frac{y_0x_{-1}}{A}$$

$$y_{15}=\max\left\{\frac{A}{y_{13}},\frac{x_{14}}{y_{13}}\right\}=\max\left\{A^3x_0,\frac{x_0y_{-1}}{A}\right\}=A^3x_0$$

$$x_{16}=\max\left\{\frac{A}{x_{14}},\frac{y_{15}}{x_{14}}\right\}=\max\left\{\frac{A^4}{y_{-1}},\frac{A^6x_0}{y_{-1}}\right\}=\frac{A^4}{y_{-1}} \qquad y_{16}=\max\left\{\frac{A}{y_{14}},\frac{x_{15}}{y_{14}}\right\}=\max\left\{\frac{A^4}{x_{-1}},\frac{A^6y_0}{x_{-1}}\right\}=\frac{A^4}{x_{-1}}$$

$$x_{17}=\max\left\{\frac{A}{x_{15}},\frac{y_{16}}{x_{15}}\right\}=\max\left\{\frac{A^2}{y_0x_{-1}},\frac{A^5}{y_0x_{-1}^{\ 2}}\right\}=\frac{A^5}{y_0x_{-1}^{\ 2}} \qquad y_{17}=\max\left\{\frac{A}{y_{15}},\frac{x_{16}}{y_{15}}\right\}=\max\left\{\frac{A^2}{x_0y_{-1}},\frac{A^5}{x_0y_{-1}^{\ 2}}\right\}=\frac{A}{x_0y_{-1}}$$

$$x_{18}=\max\left\{\frac{A}{x_{16}},\frac{y_{17}}{x_{16}}\right\}=\max\left\{\frac{y_{-1}}{A^3},\frac{1}{x_0A^3}\right\}=\frac{1}{x_0A^3} \qquad y_{18}=\max\left\{\frac{A}{y_{16}},\frac{x_5}{y_4}\right\}=\max\left\{\frac{x_{-1}}{A^3},\frac{A}{y_0x_{-1}}\right\}=\frac{A}{y_0x_{-1}}$$

$$x_{19}=\max\left\{\frac{A}{x_{15}},\frac{y_{16}}{x_{15}}\right\}=\max\left\{\frac{y_0x_{-1}^{\ 2}}{A^4},\frac{x_{-1}}{A^4}\right\}=\frac{x_{-1}}{A^4} \qquad y_{19}=\max\left\{\frac{A}{y_{15}},\frac{x_{16}}{y_{15}}\right\}=\max\left\{x_0y_{-1},\frac{y_{-1}}{A^4}\right\}=\frac{y_{-1}}{A^4}$$

$$x_{20}=\max\left\{\frac{A}{x_{16}},\frac{y_{17}}{x_{16}}\right\}=\max\left\{A^4x_0,\frac{x_0y_{-1}}{A}\right\}=A^4x_0 \qquad y_{20}=\max\left\{\frac{A}{y_{16}},\frac{x_5}{y_4}\right\}=\max\left\{y_0x_{-1},\frac{y_0x_{-1}^{\ 2}}{A^5}\right\}=\frac{y_0x_{-1}^{\ 2}}{A^5}$$

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elde edilir.

Teorem 4.12:Eğer A<1 ise $x_{n+1}=\max\left\{\frac{A}{x_{n-1}},\frac{y_n}{x_{n-1}}\right\}; y_{n+1}=\max\left\{\frac{A}{y_{n-1}},\frac{x_n}{y_{n-1}}\right\}$ denkleminin $(x_n; y_n)$ çözümleri $y_{-1}<x_{-1}<x_0<y_0<A<1$, $y_{-1}<x_{-1}<y_0<x_0<A<1$ başlangıç şartlarına göre aşağıdaki şekildedir.

X_n ÇÖZÜMLERİ

$$x_{10n+1}=\frac{A^{2n+1}}{x_{-1}};$$

$$x_{10n+2}=\frac{A}{x_0y_{-1}};$$

n=0,1 için $x_{10n+3}=\frac{1}{A^{2n}y_0}$, $n\geq 2$ için $x_{10n+3}=\frac{A}{x_{-1}y_0}$;

$$x_{10n+4}=\frac{y_{-1}}{A^{2n+1}};$$

n=0 için $x_{10n+5}=A\cdot y_0$, $n\geq 2$ için $x_{10n+5}=\frac{x_{-1}y_0}{A}$;

$$x_{10n+6}=\frac{A^{2n+2}}{y_{-1}};$$

n=0 için $x_{10n+7}=\frac{A}{x_{-1}y_0}$, $n\geq 1$ için $x_{10n+7}=\frac{A^{2n+3}}{x_{-1}^2 y_0}$;

$$x_{10n+8}=\frac{1}{A^{2n+1}x_0};$$

$$x_{10n+9}=\frac{x_{-1}}{A^{2n+2}};$$

$$x_{10n+10}=A^{2n+2}\cdot x_0,$$

Y_n ÇÖZÜMLERİ

$$y_{10n+1}=\frac{A^{2n+1}}{y_{-1}};$$

n=0,1 için $y_{10n+2}=\frac{A}{x_{-1}y_0}$, $n\geq 2$ için $y_{10n+2}=\frac{A^{2n+2}}{x_{-1}^2 y_0}$;

$$y_{10n+3}=\frac{1}{A^{2n}x_0};$$

$$y_{10n+4}=\frac{x_{-1}}{A^{2n+1}};$$

$$y_{10n+5}=A^{2n+1}\cdot x_0;$$

$$y_{10n+6}=\frac{A^{2n+2}}{x_{-1}};$$

$$y_{10n+7}=\frac{A}{x_0 y_{-1}};$$

n=0 için $y_{10n+8}=\frac{1}{Ay_0}$, $n\geq 1$ için $y_{10n+8}=\frac{A}{x_{-1}y_0}$;

$$y_{10n+9}=\frac{y_{-1}}{A^{2n+2}};$$

n=0 için $y_{10n+10}=A^2\cdot y_0$, $n\geq 1$ için $y_{10n+10}=\frac{x_{-1}^2 y_0}{A^{2n+3}}$;

İspat:

Bu teoremin ispatını n nin değerleri için gösterelim.

$$x_1 = \max\left\{\frac{A}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \frac{A}{x_{-1}} \qquad y_1 = \max\left\{\frac{A}{y_{-1}}, \frac{x_0}{y_{-1}}\right\} = \frac{A}{y_{-1}}$$

$$x_2 = \max\left\{\frac{A}{x_0}, \frac{y_1}{x_0}\right\} = \max\left\{\frac{A}{x_0}, \frac{A}{y_{-1}x_0}\right\} = \frac{A}{y_{-1}x_0} \qquad y_2 = \max\left\{\frac{A}{y_0}, \frac{x_1}{y_0}\right\} = \max\left\{\frac{A}{y_0}, \frac{A}{x_{-1}y_0}\right\} = \frac{A}{x_{-1}y_0}$$

$$x_3 = \max\left\{\frac{A}{x_0}, \frac{y_1}{x_0}\right\} = \max\left\{x_{-1}, \frac{1}{y_0}\right\} = \frac{1}{y_0} \qquad y_3 = \max\left\{\frac{A}{y_1}, \frac{x_2}{y_1}\right\} = \max\left\{\frac{Ay_{-1}}{A}, \frac{Ay_{-1}}{x_0Ay_{-1}}\right\} = \frac{1}{x_0}$$

$$x_4 = \max\left\{\frac{A}{x_2}, \frac{y_3}{x_2}\right\} = \max\left\{x_0y_{-1}, \frac{y_{-1}}{A}\right\} = \frac{y_{-1}}{A} \qquad y_4 = \max\left\{\frac{A}{y_2}, \frac{x_3}{y_2}\right\} = \max\left\{y_0x_{-1}, \frac{x_{-1}}{A}\right\} = \frac{x_{-1}}{A}$$

$$x_5 = \max\left\{\frac{A}{x_3}, \frac{y_4}{x_3}\right\} = \max\left\{A\,y_0, x_{-1}\frac{y_0}{A}\right\} = A\,y_0 \qquad y_5 = \max\left\{\frac{A}{y_3}, \frac{x_4}{y_3}\right\} = \max\left\{A\,x_0, y_{-1}\frac{x_0}{A}\right\} = A\,x_0$$

$$x_6 = \max\left\{\frac{A}{x_4}, \frac{y_5}{x_4}\right\} = \max\left\{\frac{A^2}{y_{-1}}, \frac{A^2x_0}{y_{-1}}\right\} = \frac{A^2}{y_{-1}} \qquad y_6 = \max\left\{\frac{A}{y_4}, \frac{x_5}{y_4}\right\} = \max\left\{\frac{A^2}{x_{-1}}, \frac{A^2y_0}{x_{-1}}\right\} = \frac{A^2}{x_{-1}}$$

$$x_7 = \max\left\{\frac{A}{x_{-1}}, \frac{y_0}{x_{-1}}\right\} = \left\{\frac{1}{y_0}, \frac{A}{x_{-1}y_0}\right\} = \frac{A}{x_{-1}y_0} \qquad y_7 = \max\left\{\frac{A}{y_5}, \frac{x_6}{y_5}\right\} = \left\{\frac{1}{x_0}, \frac{A}{x_0y_{-1}}\right\} = \frac{A}{x_0y_{-1}}$$

$$x_8 = \max\left\{\frac{A}{x_6}, \frac{y_7}{x_6}\right\} = \max\left\{\frac{y_{-1}}{A}, \frac{1}{Ax_0}\right\} = \frac{1}{Ax_0} \qquad y_8 = \max\left\{\frac{A}{y_6}, \frac{x_7}{y_6}\right\} = \max\left\{\frac{x_{-1}}{A}, \frac{1}{Ay_0}\right\} = \frac{1}{Ay_0}$$

$$x_9 = \max\left\{\frac{A}{x_7}, \frac{y_8}{x_7}\right\} = \max\left\{y_0x_{-1}, \frac{x_{-1}}{A^2}\right\} = \frac{x_{-1}}{A^2} \qquad y_9 = \max\left\{\frac{A}{y_7}, \frac{x_8}{y_7}\right\} = \left\{x_0y_{-1}, \frac{y_{-1}}{A^2}\right\} = \frac{y_{-1}}{A^2}$$

$$x_{10} = \max\left\{\frac{A}{x_8}, \frac{y_9}{x_8}\right\} = \max\left\{A^2x_0, \frac{Ax_0y_{-1}}{A^2}\right\} = A^2x_0 \qquad y_{10} = \max\left\{\frac{A}{y_8}, \frac{x_9}{y_8}\right\} = \max\left\{A^2y_0, \frac{Ay_0x_{-1}}{A^2}\right\} = A^2y_0$$

$$x_{11} = \max\left\{\frac{A}{x_9}, \frac{y_{10}}{x_9}\right\} = \max\left\{\frac{A^3}{x_{-1}}, \frac{A^4y_0}{x_{-1}}\right\} = \frac{A^3}{x_{-1}} \qquad y_{11} = \max\left\{\frac{A}{y_9}, \frac{x_{10}}{y_9}\right\} = \max\left\{\frac{A^3}{y_{-1}}, \frac{A^4x_0}{y_{-1}}\right\} = \frac{A^3}{y_{-1}}$$

$$x_{12} = \max\left\{\frac{A}{x_{10}}, \frac{y_{11}}{x_{10}}\right\} = \max\left\{\frac{1}{Ax_0}, \frac{A}{x_0y_{-1}}\right\} = \frac{A}{x_0y_{-1}} \qquad y_{12} = \max\left\{\frac{A}{y_{10}}, \frac{x_{11}}{y_{10}}\right\} = \max\left\{\frac{1}{A^2y_0}, \frac{A}{y_0x_{-1}}\right\} = \frac{A}{y_0x_{-1}}$$

$$x_{13} = \max\left\{\frac{A}{x_{11}}, \frac{y_{12}}{x_{11}}\right\} = \left\{\frac{x_{-1}}{A^2}, \frac{1}{A^2y_0}\right\} = \frac{1}{A^2y_0} \qquad y_{13} = \max\left\{\frac{A}{y_{11}}, \frac{x_{12}}{y_{11}}\right\} = \left\{\frac{y_{-1}}{A^2}, \frac{1}{A^2x_0}\right\} = \frac{1}{A^2x_0}$$

$$x_{14} = \max\left\{\frac{A}{x_{12}}, \frac{y_{13}}{x_{12}}\right\} = \max\left\{x_0y_{-1}, \frac{y_{-1}}{A^3}\right\} = \frac{y_{-1}}{A^3} \qquad y_{14} = \max\left\{\frac{A}{y_{12}}, \frac{x_{13}}{y_{12}}\right\} = \max\left\{y_0x_{-1}, \frac{x_{-1}}{A^3}\right\} = \frac{x_{-1}}{A^3}$$

$$x_{15}=\max\left\{\frac{A}{x_{13}},\frac{y_{14}}{x_{13}}\right\}=\max\left\{A^3\,\mathrm{y}_0,\frac{y_0x_{-1}}{A}\right\}=\frac{y_0x_{-1}}{A} \qquad \mathrm{y}_{15}=\max\left\{\frac{A}{y_{13}},\frac{x_{14}}{y_{13}}\right\}=\max\left\{\mathrm{A}^3\mathrm{x}_0,\frac{x_0y_{-1}}{A}\right\}=A^3x_0$$

$$x_{16}=\max\left\{\frac{A}{x_{14}},\frac{y_{15}}{x_{14}}\right\}=\max\left\{\frac{A^4}{y_{-1}},\frac{A^6x_0}{y_{-1}}\right\}=\frac{A^4}{y_{-1}} \qquad \mathrm{y}_{16}=\max\left\{\frac{A}{y_{14}},\frac{x_{15}}{y_{14}}\right\}=\max\left\{\frac{A^4}{x_{-1}},\frac{A^6y_0}{x_{-1}}\right\}=\frac{A^4}{x_{-1}}$$

$$x_{17}=\max\left\{\frac{A}{x_{15}},\frac{y_{16}}{x_{15}}\right\}=\max\left\{\frac{A^2}{y_0x_{-1}},\frac{A^5}{y_0x_{-1}{}^2}\right\}=\frac{A^5}{y_0x_{-1}{}^2} \qquad \mathrm{y}_{17}=\max\left\{\frac{A}{y_{15}},\frac{x_{16}}{y_{15}}\right\}=\max\left\{\frac{\mathrm{A}^2}{x_0y_{-1}},\frac{A^5}{x_0y_{-1}{}^2}\right\}=\frac{A}{x_0y_{-1}}$$

$$x_{18}=\max\left\{\frac{A}{x_{16}},\frac{y_{17}}{x_{16}}\right\}=\max\left\{\frac{y_{-1}}{A^3},\frac{1}{x_0A^3}\right\}=\frac{1}{x_0A^3} \qquad \mathrm{y}_{18}=\max\left\{\frac{A}{y_{16}},\frac{x_5}{y_4}\right\}=\max\left\{\frac{x_{-1}}{A^3},\frac{A}{y_0x_{-1}}\right\}=\frac{A}{y_0x_{-1}}$$

$$x_{19}=\max\left\{\frac{A}{x_{15}},\frac{y_{16}}{x_{15}}\right\}=\max\left\{\frac{y_0x_{-1}{}^2}{A^4},\frac{x_{-1}}{A^4}\right\}=\frac{x_{-1}}{A^4} \qquad \mathrm{y}_{19}=\max\left\{\frac{A}{y_{15}},\frac{x_{16}}{y_{15}}\right\}=\max\left\{x_0y_{-1},\frac{y_{-1}}{A^4}\right\}=\frac{y_{-1}}{A^4}$$

$$x_{20}=\max\left\{\frac{A}{x_{16}},\frac{y_{17}}{x_{16}}\right\}=\max\left\{A^4x_0,\frac{x_0y_{-1}}{A}\right\}=A^4x_0 \qquad \mathrm{y}_{20}=\max\left\{\frac{A}{y_{16}},\frac{x_5}{y_4}\right\}=\max\left\{y_0x_{-1},\frac{y_0x_{-1}{}^2}{A^5}\right\}=\frac{y_0x_{-1}{}^2}{A^5}$$

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elde edilir.

3.2.ÖRNEKLER

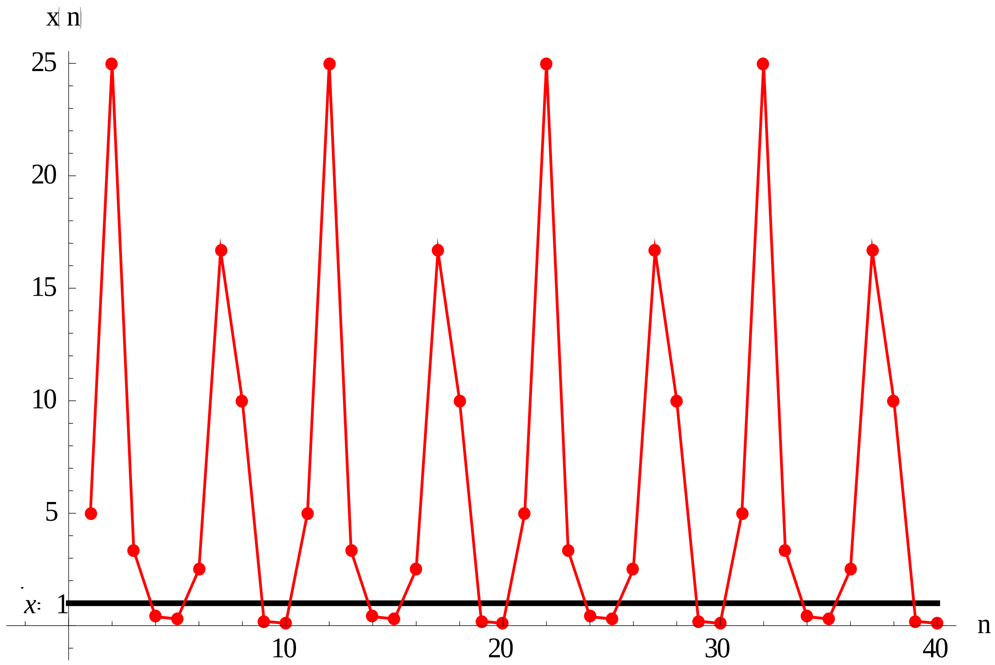
ÖRNEK 1: Başlangıç şartları Lemma 1 ve Teorem 1 dekine uygun bir şekilde seçilirse

A=1 ,x[-1] = 0.2; x[0] = 0.1; y[-1] = 0.4; y[0] = 0.3;

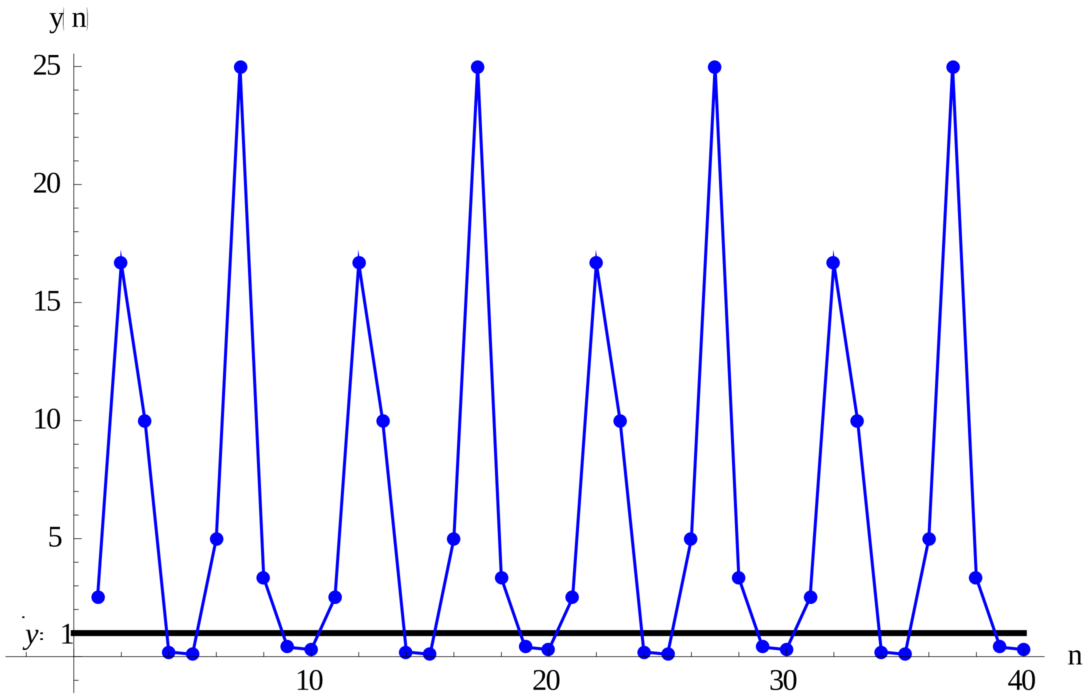
x(n)={5.,25.,3.33333,0.4,0.3,2.5,16.6667,10.,0.2,0.1,
5.,25.,3.33333,0.4,0.3,2.5,16.6667,10.,0.2,0.1, ...}

y(n)={2.5,16.6667,10.,0.2,0.1,5.,25.,3.33333,0.4,0.3,
2.5,16.6667,10.,0.2,0.1,5.,25.,3.33333,0.4,0.3, ...}

çözümleri elde edilir ve çözümlerin grafikleri aşağıda gösterilmiştir.



Şekil 1. x(n) çözümlerinin grafiği.



Şekil 2. y(n) çözümlerinin grafiği.

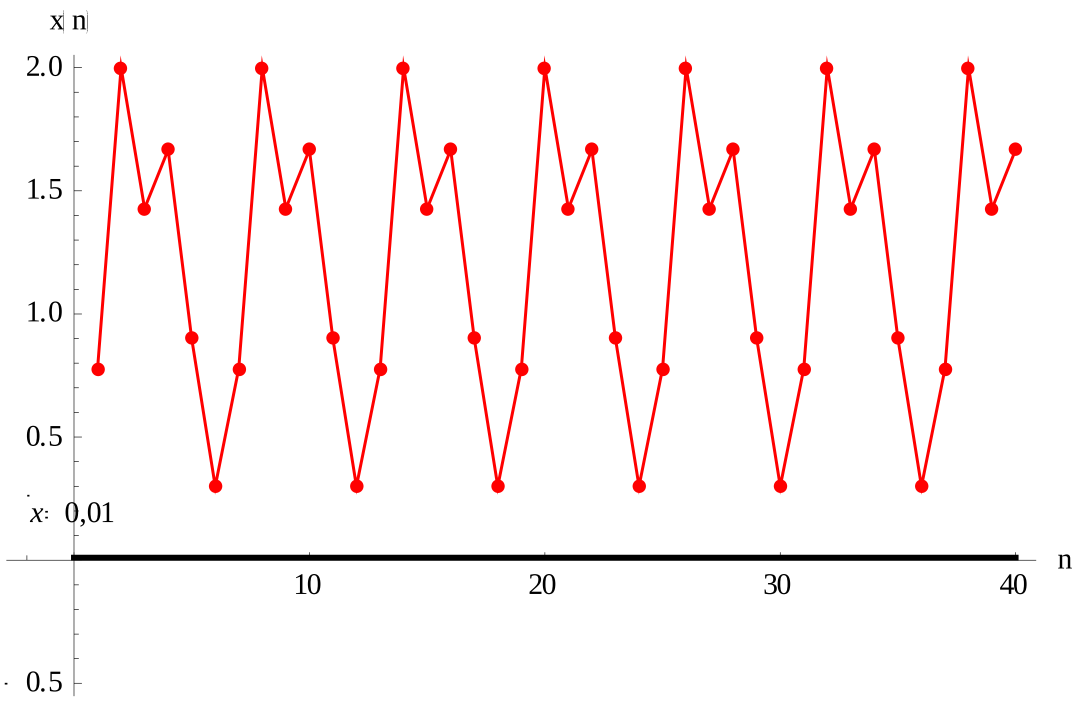
ÖRNEK **2:** Başlangıç şartları Lemma 2 ve Teorem 2 dekine uygun bir şekilde seçilirse

A = 0.1 $x[-1] = 0.9$; $x[0] = 0.3$; $y[-1] = 0.5$; $y[0] = 0.7$

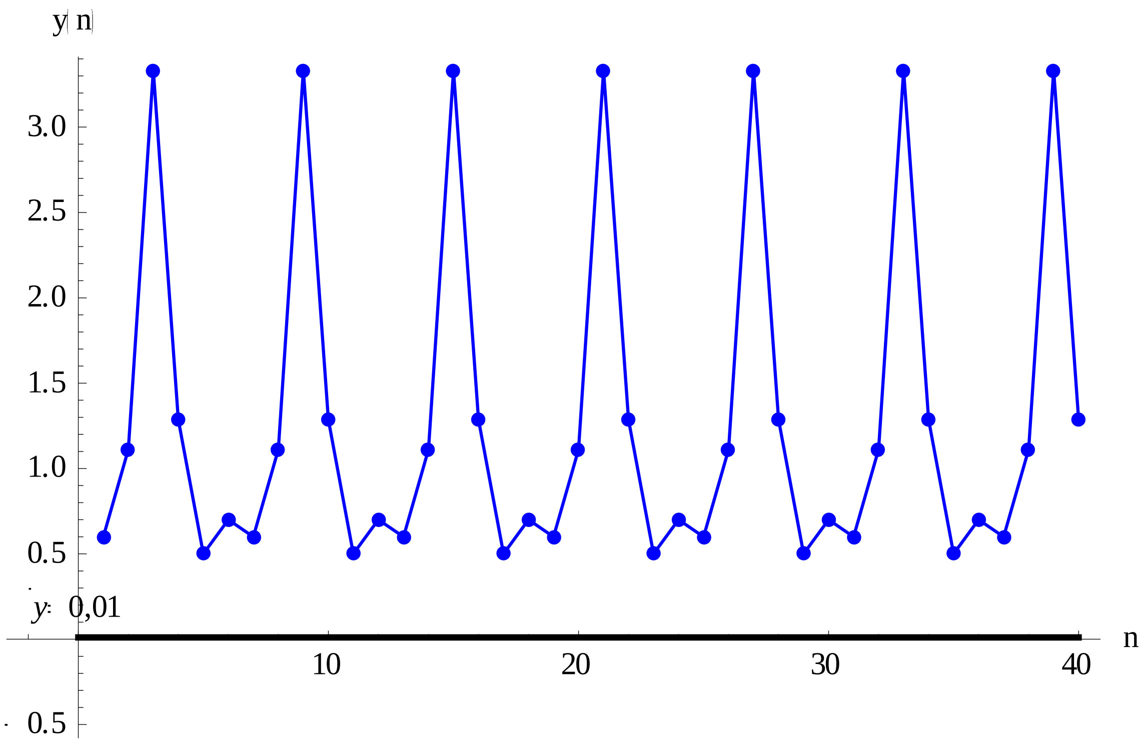
x(n)={0.777778,2.,1.42857,1.66667,0.9,0.3,0.777778,2.,
1.42857,1.66667,0.9,0.3,0.777778,2.,1.42857,
1.66667,0.9,0.3,0.777778,2.,1.42857,1.66667,0.9,
0.3,0.777778,2.,1.42857,1.66667,0.9,0.3,0.777778,2.,
1.42857,1.66667,0.9,0.3,0.777778,2.,1.42857,1.66667}

y(n)={0.6,1.11111,3.33333,1.28571,0.5,0.7,0.6,1.11111,
3.33333,1.28571,0.5,0.7,0.6,1.11111,3.33333,
1.28571,0.5,0.7,0.6,1.11111,3.33333,1.28571,0.5,
0.7,0.6,1.11111,3.33333,1.28571,0.5,0.7,0.6,1.11111,
3.33333,1.28571,0.5,0.7,0.6,1.11111,3.33333,1.28571}

çözümleri elde edilir ve çözümlerin grafikleri aşağıda gösterilmiştir.



Şekil 3. x(n) çözümlerinin grafiği.



Şekil 4. y(n) çözümlerinin grafiği.

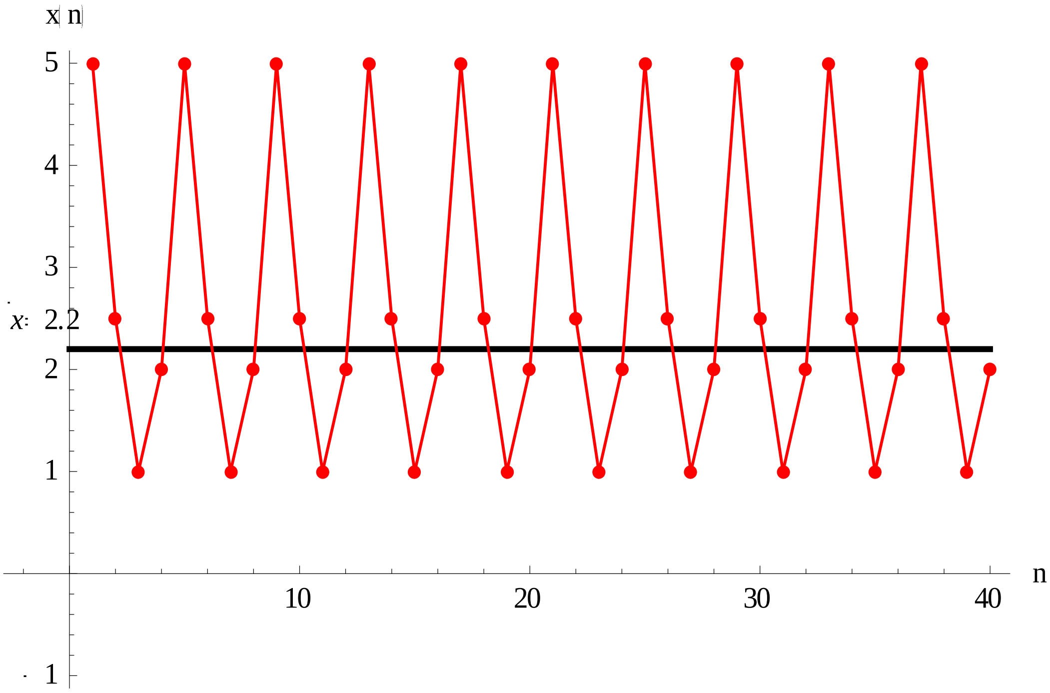
ÖRNEK **3:** Başlangıç şartları Teorem 2'ye uygun bir şekilde seçilirse

$x[-1] = 1;\ x[0] = 2;\ y[-1] = 3;\ y[0] = 4;$ A = 5

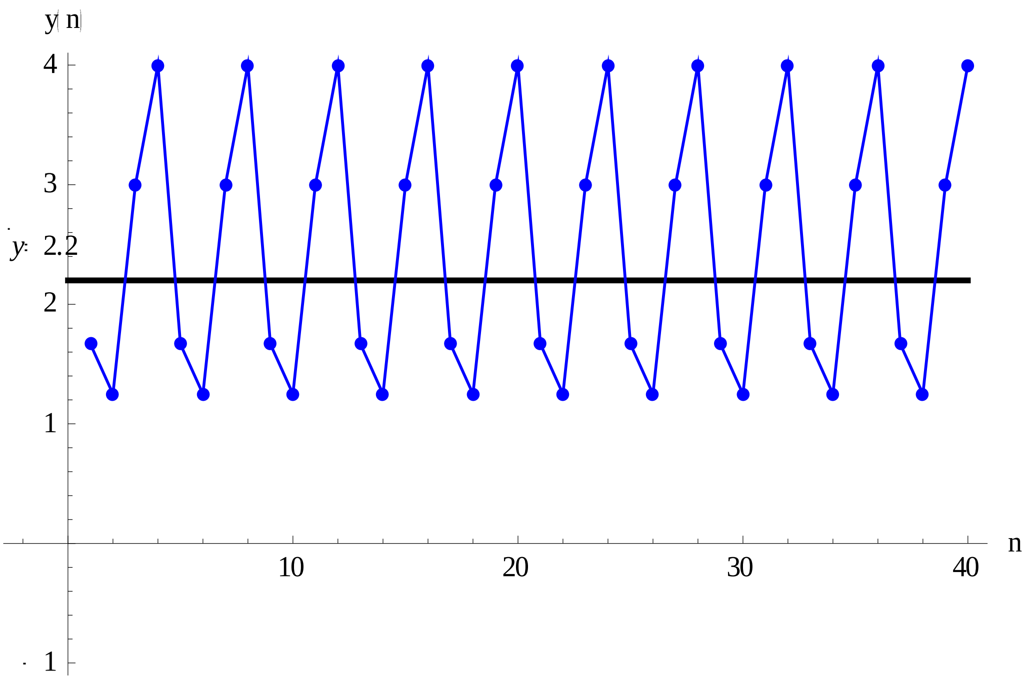
x(n)={5,2.5,1,2, 5,2.5,1,2, 5,2.5,1,2...}

y(n)={1.66667,1.25,3.,4., 1.66667,1.25,3.,4., 1.66667,1.25,3.,4...}

çözümleri elde edilir ve çözümlerin grafikleri aşağıda gösterilmiştir.



Şekil 5. x(n) çözümlerinin grafiği.



Şekil 6. y(n) çözümlerinin grafiği.

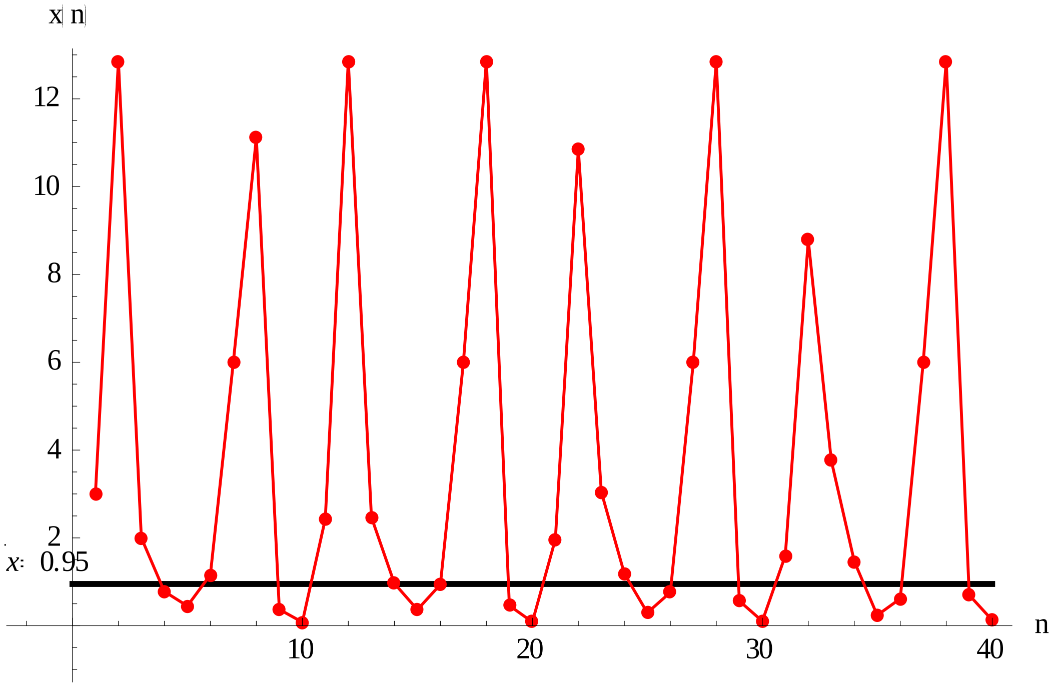
ÖRNEK **4:** Başlangıç şartları Teorem 4.1'e $x_0 < x_{-1} < y_0 < y_{-1} < A < 1$ uygun bir şekilde seçilirse

A <1 ve $x[-1] = 0.3$; $x[0] = 0.1$; $y[-1] = 0.7$; $y[0] = 0.5$; A =0.9

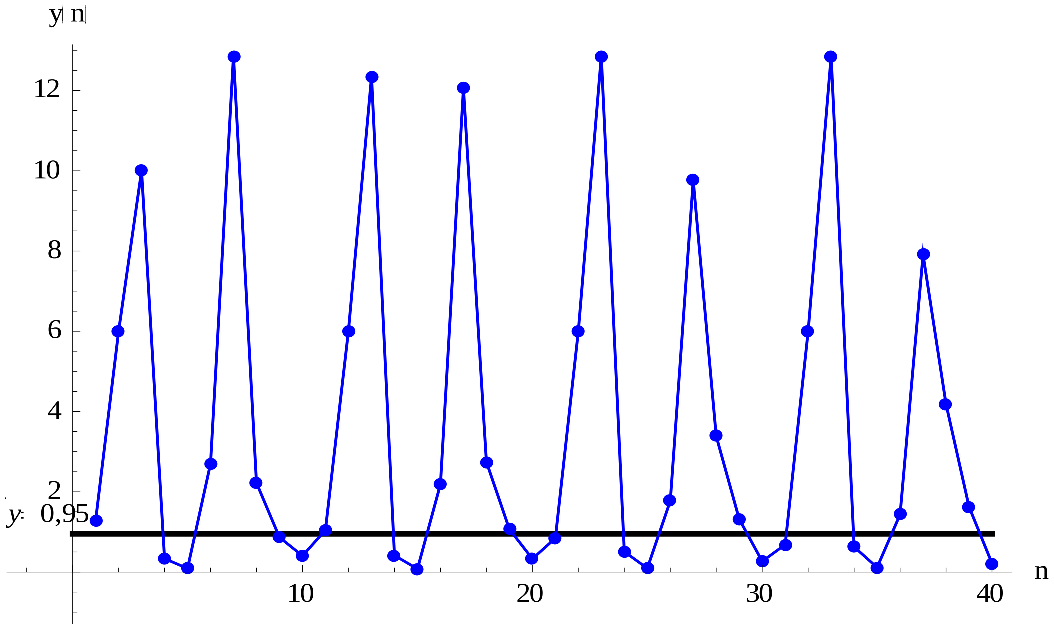
x(n)={3.,12.8571,2.,0.777778,0.45,1.15714,6.,11.1111,0.37037,
0.081,2.43,12.8571,2.46914,0.960219,0.3645,0.937286,
6.,12.8571,0.457247,0.0829819,1.9683,10.8457,3.04832,
1.18546,0.295245,0.759201,6.,12.8571,0.564503,0.102447}

y(n)={1.28571,6.,10.,0.333333,0.09,2.7,12.8571,2.22222,0.864198,
0.405,1.04143,6.,12.3457,0.411523,0.0777778,2.187,12.0508,
2.74348,1.06691,0.32805,0.843557,6.,12.8571,0.508053,
0.0922021,1.77147,9.76116,3.38702,1.31717,0.265721}

çözümleri elde edilir ve çözümlerin grafikleri aşağıda gösterilmiştir.



Şekil 7. x(n) çözümlerinin grafiği.



Şekil 8. y(n) çözümlerinin grafiği.

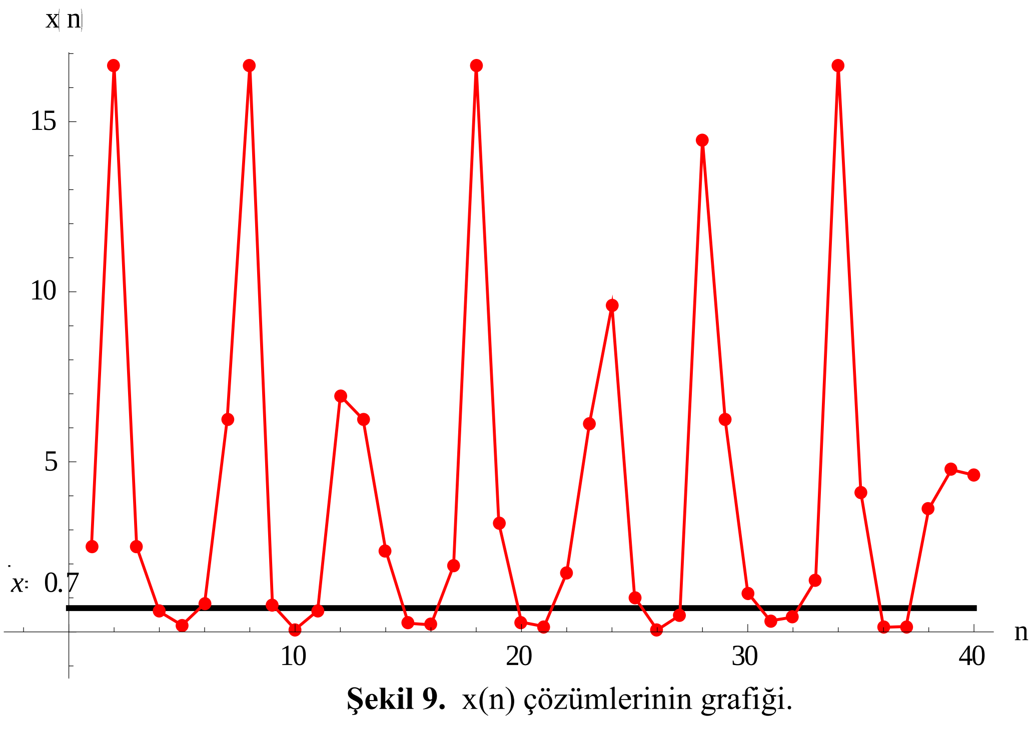
ÖRNEK **5:** Başlangıç şartları Teorem 4.2'e $x_0 < x_{-1} < y_{-1} < y_0 < A < 1$ uygun bir şekilde seçilirse

A <1 ve $x[-1]=0.2$; $x[0]=0.1$; $y[-1]=0.3$; $y[0]=0.4$; A=0.5

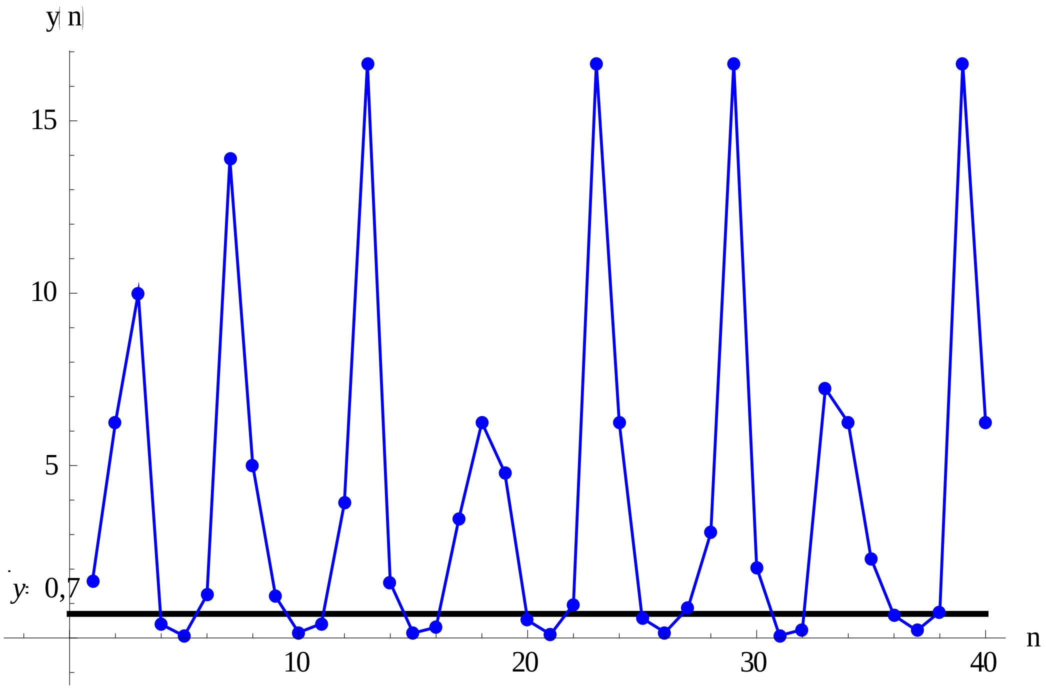
x(n)={2.5,16.6667,2.5,0.6,0.2,0.833333,6.25,16.6667,0.8,0.072,0.625,
6.94444,6.25,2.4,0.256,0.208333,1.95313,16.6667,3.2,0.288,0.16,
1.73611,6.10352,9.6,1.024,0.06,0.488281,14.4676,6.25,1.152,0.32768,
0.434028,1.52588,16.6667,4.096,0.13824,0.16,3.6169,4.76837,4.608...}

y(n)={1.66667,6.25,10.,0.4,0.06,1.25,13.8889,5.,1.2,0.16,0.416667,
3.90625,16.6667,1.6,0.144,0.3125,3.47222,6.25,4.8,0.512,0.104167,
0.976563,16.6667,6.25,0.576,0.16384,0.868056,3.05176,16.6667,2.048,
0.06912,0.244141,7.2338,6.25,2.304,0.65536,0.217014,0.762939,16.6667,...}

çözümleri elde edilir ve çözümlerin grafikleri aşağıda gösterilmiştir.



Şekil 9. x(n) çözümlerinin grafiği.



Şekil 10. y(n) çözümlerinin grafiği.

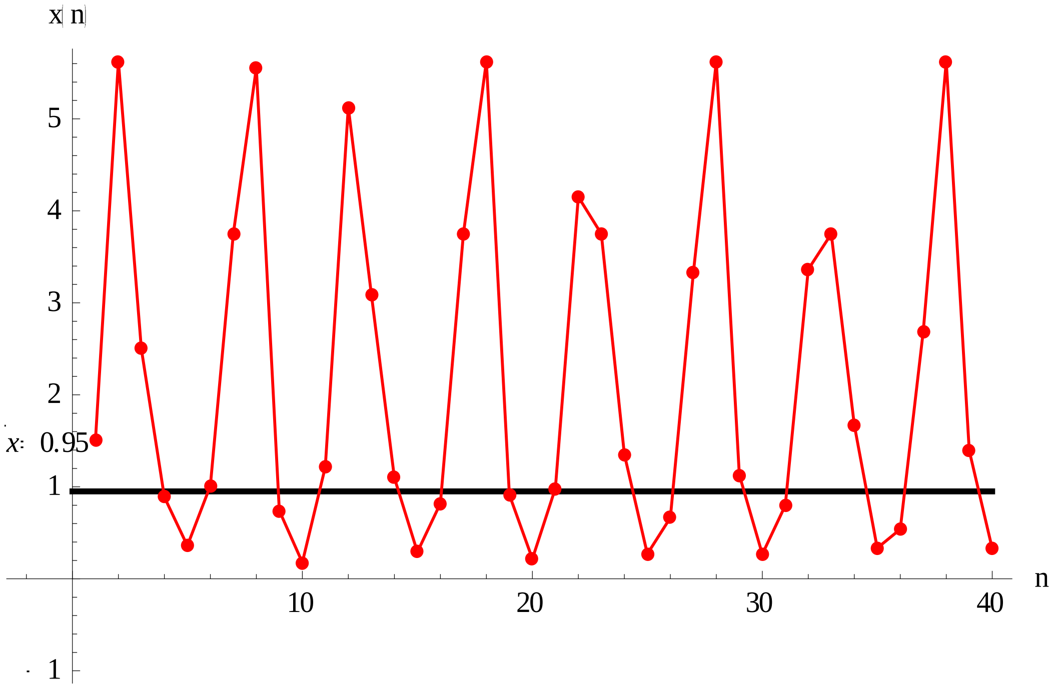
ÖRNEK **6:** Başlangıç şartları Teorem 4.3'e $x_0 < y_0 < x_{-1} < y_{-1} < A < 1$ uygun bir şekilde seçilirse

A =0.9 ve $x[-1] = 0.6$; $x[0] = 0.2$; $y[-1] = 0.8$; $y[0] = 0.4$;

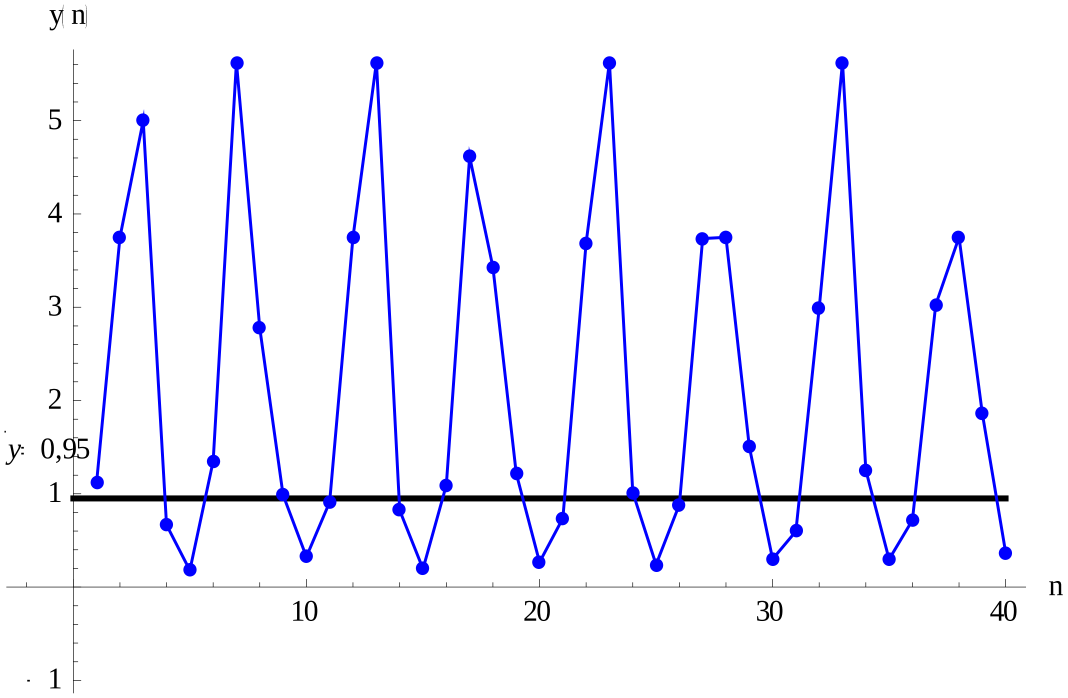
x(n)={1.5,5.625,2.5,0.888889,0.36,1.0125,3.75,5.55556,0.740741,
0.177778,1.215,5.12578,3.08642,1.09739,0.2916,0.820125,
3.75,5.625,0.914495,0.216769,0.98415,4.15188,3.75,1.35481,
0.270961,0.664301,3.32151,5.625,1.12901,0.267616,0.797162,3.36303,
3.75,1.6726,0.33452,0.538084,2.69042,5.625,1.39383,0.33039,...}

y(n)={1.125,3.75,5.,0.666667,0.18,1.35,5.625,2.77778,0.987654,
0.324,0.91125,3.75,5.625,0.823045,0.195092,1.0935,4.6132,3.42936,
1.21933,0.266667,0.738112,3.69056,5.625,1.01611,0.240855,0.885735,
3.73669,3.75,1.50534,0.301068,0.597871,2.98936,5.625,1.25445,0.297351,...}

çözümleri elde edilir ve çözümlerin grafikleri aşağıda gösterilmiştir.



Şekil 11. x(n) çözümlerinin grafiği.



Şekil 12. y(n) çözümlerinin grafiği.

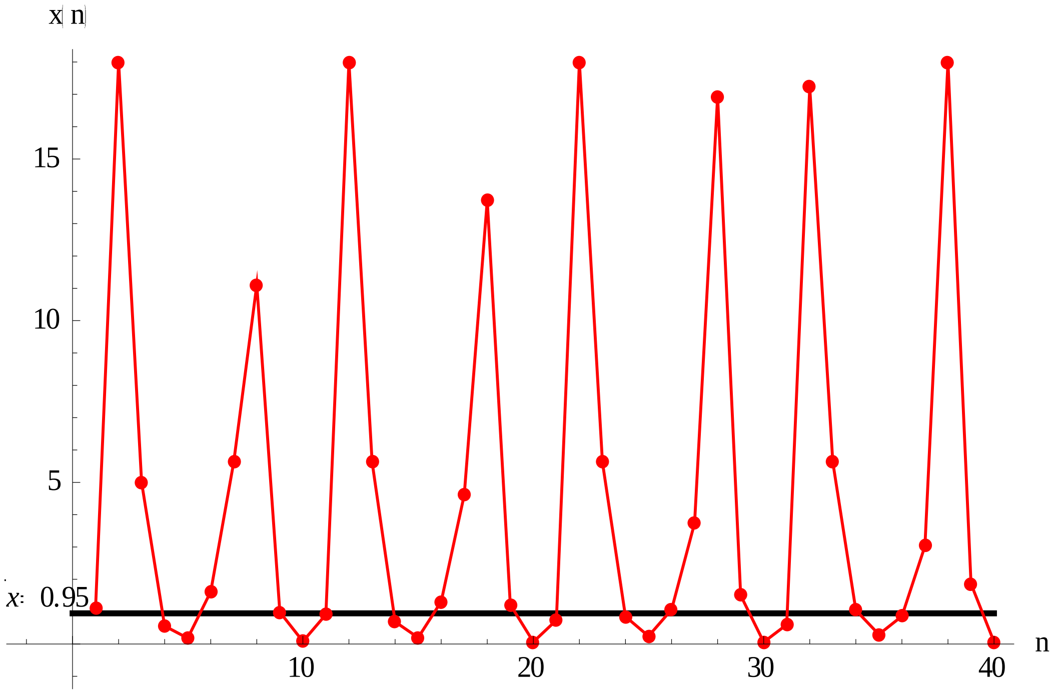
ÖRNEK **7:** Başlangıç şartları Teorem 4.4'e $x_0 < y_0 < y_{-1} < x_{-1} < A < 1$ uygun bir şekilde seçilirse

A =0.9 ve $x[-1] = 0.8$; $x[0]$ =0.1; $y[-1] = 0.5$; $y[0]$ =0.2;

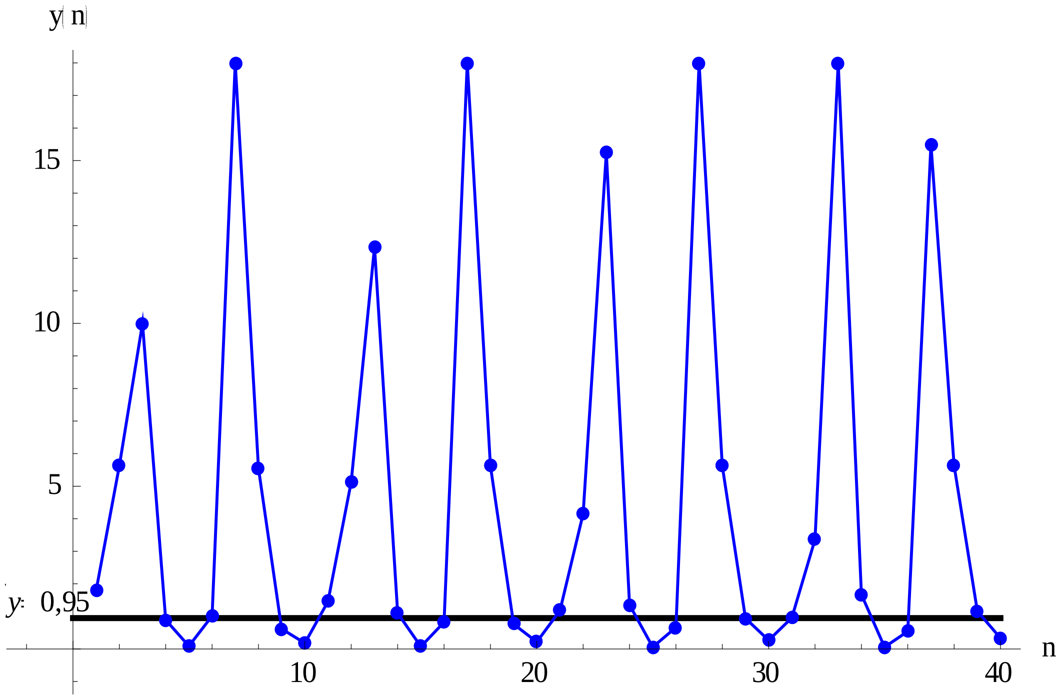
x(n)={1.125,18.,5.,0.555556,0.18,1.62,5.625,11.1111,0.987654,0.081,
0.91125,18.,5.625,0.685871,0.195092,1.3122,4.6132,13.7174,1.21933,
0.06561,0.738112,18.,5.625,0.846754,0.240855,1.06288,3.73669,16.9351,
1.50534,0.0555556,0.597871,17.2187,5.625,1.04538,0.297351,0.860934,...}

y(n)={1.8,5.625,10.,0.888889,0.09,1.0125,18.,5.55556,0.617284,0.177778,1.458,
5.12578,12.3457,1.09739,0.0729,0.820125,18.,5.625,0.762079,0.216769,
1.18098,4.15188,15.2416,1.35481,0.059049,0.664301,18.,5.625,0.940838,0.267616,
0.956594,3.36303,18.,1.6726,0.0580764,0.538084,15.4968,5.625,1.16153,0.33039,...}

çözümleri elde edilir ve çözümlerin grafikleri aşağıda gösterilmiştir.



Şekil 13. x(n) çözümlerinin grafiği.



Şekil 14. y(n) çözümlerinin grafiği.

SONUÇ VE ÖNERİLER

Bu çalışmada, sıfırdan farklı reel sayılar olan A, x_0, x_{-1}, y_0, y_{-1} başlangıç şartları için $x_{n+1} = \max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_{n-1}}\right\}; y_{n+1} = \max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_{n-1}}\right\}$ maksimumlu fark denklem sisteminin çözümlerinin davranışları, farklı durumlar için genel çözümleri elde edilmiş ve periyodikliği incelenmiştir.

Bu maksimumlu fark denklem sisteminde katsayılar değiştirilerek yeni maksimumlu fark denklem sistemleri oluşturulabilir. Aynı zamanda maksimumlu fark denklem sistemi genelleştirilerek çözümü, çözümlerin davranışları ve periyodikliği incelenebilir.

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Türkçe

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